Litigation with Symmetric Bargaining and Two-Sided Incomplete Information

Daniel Friedman
University of California, Santa Cruz

Donald Wittman
University of California, Santa Cruz

We construct game-theoretic foundations for bargaining in the shadow of a trial. Plaintiff and defendant both have noisy signals of a common-value trial judgment and make simultaneous offers to settle. If the offers cross, they settle on the average offer; otherwise, both litigants incur an additional cost and the judgment is imposed at trial. We obtain an essentially unique Nash equilibrium and characterize its conditional trial probabilities and judgments. Some of the results are intuitive. For example, an increase in trial cost (or a decrease in the range of possible outcomes) reduces the probability of a trial. We obtain a precise nonlinear expression for the relationship. Other results reverse findings from previous literature. For example, trials are possible even when the defendant's signal indicates a higher potential judgment than the plaintiff's signal, and when trial costs are low, middling cases (rather than extreme cases) are more likely to settle.

1. Introduction

Seminal work on bargaining in the shadow of a trial by Landes (1971), Gould (1973), Shavell (1982), and others assumed that the participants had differential expectations. Later writers criticized this literature for not having the proper game-theoretic underpinnings. The main points of this criticism were that these early authors (1) did not explicitly model how each side would take into account that the other side had a different but equally valid estimate of the trial outcome, (2) assumed that a settlement would take place if and only if the plaintiff expectations were below the defendant's expectations, and (3) did not consider strategic behavior by the litigants where one or both would trade-off probability of a settlement for an increased share of the surplus.[1] These criticisms are valid, but the intuition behind the early work—that the litigants have differential expectations about the trial outcome—is compelling. In this

We would like to thank Lucian Bebchuk, Huiban Yan, seminar participants at Stanford and University of California, Santa Cruz, and the referees for helpful comments.

1. These aforementioned issues arise whether or not the litigants share a common prior.

The Journal of Law, Economics, & Organization, Vol. 23, No. 1,
doi:10.1093/jleo/ewm004
Advance Access publication August 23, 2006

article, we provide a game-theoretic foundation for this intuition. In the process, we derive new results, the most startling being that when trial costs are low, the probability of a trial is highest when the probability of the plaintiff winning is either very high or very low.

Most recent research has solid game-theoretic foundations but employs one-sided incomplete information and one-sided offers (see, e.g., Bebchuk 1984), which seems highly unrealistic. Here, we generalize these one-sided models by considering a symmetric situation where there is two-sided incomplete information and both sides make offers. Results in this study comparing the offer curves of the plaintiff and the defendant cannot be derived from models with one-sided information and one-sided offers. As will be shown, our model also derives different results concerning the selection of cases for trial. The few studies that have considered two-sided incomplete information have not investigated such issues.

Plaintiffs and defendants have access to different information and, therefore, have different expectations about the outcome of a trial.[2] Although pretrial discovery may reduce some of this differential in information, much information cannot or will not be credibly conveyed to the other side. Examples include the nature of the argument that will be made to the jury, the questions to be presented to the opposing side's witnesses, and private information regarding the biases of the judge. Indeed, if the plaintiff communicated to the defendant that the judge scheduled to hear the case was biased in favor of the plaintiff, then the defendant would ask for a different judge. In this study, we consider two-sided incomplete information where both players take into account that their own information is partial, something that is not explicitly considered in the differential expectations literature.

In a recent review article, Daughety (2000) lists over 50 articles on litigation and settlement.[3] A natural question to ask is how the present study distinguishes itself from this intellectual thicket. As noted, the great majority of these articles deal with one-sided incomplete (asymmetric) information, for example, the plaintiff, but not the defendant, knows the actual damages. Restricting our attention to those three articles that deal with two-sided asymmetric information, our work differs from the previous works on a number of dimensions. First, the nature of the two-sided incomplete information is different. In previous articles, each side is privy to a special kind of information. For example, one side knows the extent of damages and the other side knows the probability of winning (Daughety and Reinganum 1994), or the plaintiff knows the strength of the plaintiff's case while the defendant knows the strength of the defendant's

2. Our discussion is in terms of plaintiffs and defendants in the context of a civil trial. Clearly, the same type of logic holds for prosecutors and defendants in a criminal trial.

3. There are also many articles about bargaining in the shadow of an arbitration verdict, but these articles tend to ignore the cost of going to arbitration. The cost of going to trial is important empirically, and thus, we explicitly model it here. Chatterjee's (1981) article on arbitration is an exception to the general rule. In his model, the litigants have private signals about their own costs, but, contrary to the model explored here, both sides have the same information regarding the arbitrator's decision.

case (Kennan and Wilson 1993).[4] Although there are some mathematical similarities, our analysis proceeds in directions not considered in these studies (e.g., the relationship between the demand and offer curves and the relationship between the cost of trial and the selection of cases for trial).

Another way that this work differentiates itself from the previous literature is the bargaining protocol. In virtually all the literature, one side makes an offer and then the other side observes the offer and either accepts it or rejects it, in which case there is a trial.[5] This creates vastly different outcomes, depending on which side makes the (first) offer. The problem is so serious that Daughety and Reinganum (1994) considered two models—one where the plaintiff makes the offer and the other where the defendant makes the offer.

Here, we present the following symmetric bargaining protocol. The plaintiff submits his/her demand and the defendant submits his/her offer to a third party (perhaps a computer). If the offer is greater than or equal to the demand, then there is a settlement halfway between the two; otherwise, the case proceeds to trial.[6] Our bargaining protocol is an extension of Chatterjee and Samuelson (1983). They consider a buyer and a seller, each having a private value for the good drawn from a uniform distribution. If the demand by the seller is less than the offer by the buyer, then there is a trade at the halfway point; otherwise, the seller keeps the item and the buyer keeps his/her money.

The Chatterjee-Samuelson (C-S) model plays a central role in understanding bargaining in markets even though few, if any, actual buyer-seller negotiations employ the C-S protocol. One could imagine all kinds of complicated, possibly unsolvable, "realistic" dynamic protocols, where some information may or may not be revealed in earlier rounds. The reason for the influence of the C-S protocol is that it provides a sensible reduced form for these more complicated models, and at the same time, it provides the actual solution to the particular protocol.

We believe that the justification for using the C-S protocol in the litigation sphere is at least as strong as, and possibly stronger than, the justification for using it in buyer-seller markets. Again, the protocol can be seen as a reduced form of a more complicated but unspecified haggling between the plaintiff and defendant lawyers. Second, the resolution could be R percent of the way between the plaintiff's demand and the defendant's maximal offer, where R has an expected value of 0.5 (the same extension could be applied to the C-S

4. Schweizer (1989) is the third study that deals with two-sided incomplete information, but he allows only two discrete information types, good news and bad news. Here we deal with a continuum of information types.

5. There are extended versions, where there may be multiple offers by one party as in Spier (1992).

6. One might ask why the computer does not just announce a settlement at the average of the demands and offers even if there is not an overlap or undertake a more sophisticated analysis and figure out the true observations from the litigants' demands and offers and then announce a settlement at the average of their observations. Unfortunately, that protocol provides perverse incentives, for example, the plaintiff should demand infinity.

protocol). Finally, there is an actual mechanism, employed by Cybersettle, that uses a reasonably similar protocol to resolve legal disputes.[7]

Despite the similarities to Chatterjee and Samuelson, our study differs in two fundamental ways. First as noted earlier, we are concerned with common values: the true value depends on both signals. Second, the truth can be revealed through a costly trial.

Since Daughety's review, a number of articles have been written using two-sided incomplete information either in the context of an infinite-horizon offer-and-counteroffer model or in the context of designing an efficient litigation mechanism.

The Rubinstein offer-and-counteroffer models do not fit the litigation game very well as there is usually a fixed court date, and in many situations, there is no cost of delay (or required costly bargaining) to one or both sides. Furthermore, these Rubinstein models give greater bargaining power to the side that makes the first offer. Here, we focus on games with symmetric bargaining power. Not surprisingly, the Rubinstein bargaining literature has focused on different issues from those considered here: the nature of the two-sided incomplete information is different (e.g., the cost of bargaining to the other party is unknown), and the concerns are different (e.g., the relative inefficiency of a repeated demand-and-offer game is compared to a one-time-only offer game). See Ausubel et al. (2002) for a recent review and a proof that the Chatterjee-Samuelson one-time offer game is an upper bound on the efficiency of the alternating-offer game.

The literature concerned with the design of efficient mechanisms for dispute resolution has also made use of two-sided incomplete information (but, again, the type of incomplete information is different from that considered here). Since going to court involves a social cost, one might want to devise more efficient methods that reduce the likelihood of trial (e.g., fining the litigant whose offer was further away from the court award). But these issues of mechanism design (even though they may involve some type of two-sided incomplete information) are far away from the focus of this study, which is to provide a reasonable characterization of the present arrangement and then derive the equilibrium strategies and comparative statics rather than discover a mechanism that reduces the likelihood of a trial, for example.

Our symmetric game has symmetric Nash equilibrium (NE) strategies that turn out to be piecewise linear and essentially unique. The simple equilibrium structure yields several useful results that were either not accessible or less intuitive in earlier work. For example, we derive exact expressions for the distribution of trial settlements as a function of trial costs and characterize the cases that go to trial. Priest and Klein (1984) argued that those cases where the plaintiff has close to a 50% chance of winning are more likely to go to trial

7. Cybersettle uses three rounds. The defendant provides a list of three offers and the plaintiff provides a list of three demands (neither list is made known to the other party). If the first set crosses (i.e., the offer is greater than the demand), then the case is settled halfway between. If the first set does not cross, the second set is considered. If there is no settlement on or before the third round, the case presumably goes to trial.

than cases where the plaintiff's probability of winning is closer to 0 or 1. We show that the results depend on the cost of the trial. When there are high costs of trial, the results confirm the Priest-Klein conjecture; however, when the costs of trial are low, the results are contrary.

The next section presents a general symmetric model and defines a tractable simple case. Section 3 derives precise quantitative results for the simple case. Section 4 shows that the ordinal (qualitative) properties extend to the general model (and somewhat beyond). A discussion appears in Section 5, Appendix A gives formal proofs of the propositions, and Appendix B presents derivations supporting the main arguments in Section 4.

2. The Model

The litigation game has two players, referred to as the plaintiff and the defendant, and three stages. At the preliminary stage 0, the players have common knowledge about the structure of the game, including the payoff function and the distributions of signals. In particular, both players know that the lowest possible judgment is $L \geq 0$ and the largest is $U > L$. At stage 1, the plaintiff privately observes a signal θ_p of the trial judgment drawn according to the cumulative distribution function (cdf) F^P and chooses a demand p. Simultaneously, the defendant privately observes an independent signal θ_d drawn from his/her cdf F^D and chooses an offer d. At the final stage 2, the payoffs are determined as follows. (a) If $d \geq p$, the case is settled at the average offer $[p + d]/2$. (b) If $d < p$, the demand and offer are inconsistent and the case goes to trial. Each player then incurs cost $c \geq 0$, and the judgment is $[\theta_p + \theta_d]/2$.

We consider pure strategies contingent on the realized signal. Thus, a plaintiff's strategy is a measurable function P defined on the support of $F^P \subseteq [L, U]$ that assigns the demand $p = P(\theta_p) \in [0, \infty)$ when he/she observes signal θ_p. Similarly, a defendant's strategy is a measurable function D defined on the support of $F^D \subseteq [L, U]$ that assigns the offer $d = D(\theta_d) \in (-\infty, \infty)$ when he/she observes signal θ_d.

The objective of the plaintiff is to maximize the expected payments (net of any court costs that might be incurred), conditioned on his/her realized signal θ_p and the defendant's strategy D. The payoff function for the plaintiff is

$$\Pi^P(p, \theta_p, D, F^D) = 0.5 \int_{[d \geq p]} [p + D(x)] \mathrm{d}F^D(x) + 0.5 \int_{[d<p]} [\theta_p + x - 2c] \mathrm{d}F^D(x). \quad (1)$$

The notation $[d < p]$ indicates the set of defendant signals $\{x : d = D(x) < p\}$ that the given D function maps to numbers less that the plaintiff's chosen demand p, and $[d \geq p]$ indicates the complementary signal set. Hence, the first term represents the expected payment due to a settlement and the second term represents the expected payment due to a trial judgment. A plaintiff strategy P is a *best response* to defendant strategy D, and we write $P \in \mathrm{PBR}(D)$ if, for each possible signal realization θ_p, the value $p = P(\theta_p)$ solves the problem $\max_y \Pi^P(y, \theta_d, D, F^D)$.

Similarly, the defendant's strategy D is a best response to plaintiff's strategy P, or $D \in \mathrm{DBR}(P)$, if it minimizes the expected payment (including possible court costs) at each signal realization θ_p. That is, $d = D(\theta_d)$ solves the problem $\min_x \Pi^D(x, \theta_d, P, F^P)$, where

$$\begin{aligned}\Pi^D(d, \theta_d, P, F^P) = 0.5 \int_{[d \geq p]} [d + P(y)]\mathrm{d}F^P(y) \\ + 0.5 \int_{[d<p]} [\theta_d + y + 2c]\mathrm{d}F^P(y).\end{aligned} \tag{2}$$

Of course, the notation $[d < p]$ now indicates the set of plaintiff signals $\{y : d < P(y)\}$, etc.

Definition. A Nash equilibrium (NE) of the litigation game is a strategy pair (D, P) such that $P \in \mathrm{PBR}(D)$ and $D \in \mathrm{DBR}(P)$.

Remarks.

- We have applied the framework and language used in auction theory (see Klemperer 1999 for a survey) to analyze a common-values situation with independent signals in the context of litigation. In the auction literature and here, the expected value of the average signal is the common value. In our model, the average value of the signal is equal to the common value. Since the litigants are risk neutral, their calculations of the expected trial outcome are not changed when the payoff from going to trial is $[\theta_p + \theta_d]/2 + \varepsilon$ instead of $[\theta_p + \theta_d]/2$ when ε has expected value of 0. Thus, the results still hold when the trial outcome equals the expected value of the average signal instead of when the trial outcome equals the average signal.
- We can characterize the relationship between the signals and the trial outcome in less abstract terms. Each side has private information about the strength of the case and beliefs on how the jury will react to its *own* argument. At trial, the jury sees both arguments (signals) and makes a judgment based on their net or average strength. That is, the trial outcome can be seen as the average of two different independent components of the case, where each side has seen one of the components. Of course, each component can be viewed as a signal about the outcome of the trial, and we will refer to the signals in this way.
- At first glance, the game might appear to be zero-sum because the plaintiff maximizes an objective function that resembles the objective function minimized by the defendant. However, the objective functions are different in two respects. First, the trial cost c offsets the plaintiff's receipts but increases the defendant's payment, so there is an overall gain when the case is settled. Second, with two-sided incomplete information, the expectations in the two objective functions are taken over different signal spaces, so the objective functions actually are not comparable.

Several normalizations and simplifications will be useful. First, in equilibrium analysis, we can restrict our attention to nondecreasing functions. As shown in Lemma 1 in Appendix A, if a strategy is a best response (to *any* opponent's strategy, whether or not an equilibrium strategy or even monotone), then it can be represented as a nondecreasing left-continuous function. Such functions have inverses,[8] so the integrals in equations (1) and (2) are from L to z and from z to U, where $z = D^{-1}(p)$ in equation (1) and $z = P^{-1}(d)$ in equation (2).

Next, without loss of generality we can normalize $L = 0$ and $U = 1$. This normalization can be obtained using a positive linear transformation of all variables (including cost, signals and bids) that has no effect on the optimization problems. However, the transformation has implications, discussed in Section 4, for the assumptions that the trial cost c is positive and that the plaintiff will proceed to trial if there is no settlement.

A *basic litigation game* (BLG) is a litigation game with the $L = 0$, $U = 1$ normalization, the restriction to nondecreasing strategies, and (more substantively) with $c \geq 0$ and uniform signal distributions. That is, $F^D(\theta) = F^P(\theta) = \theta$ and $\mathrm{d}F^D(\theta) = \mathrm{d}F^P(\theta) = \mathrm{d}\theta$ for all θ in [0, 1]. The uniform distribution assumption does involve loss of generality but, as discussed in Section 4, perhaps less than one might think. It is very convenient because the objective functions in the BLG take the form:

$$\Pi^P(p, \theta_p, D, F^D) = 0.5 \int_{D^{-1}(p)}^{1} [p + D(x)]\mathrm{d}x + 0.5 \int_{0}^{D^{-1}(p)} [\theta_p + x - 2c]\mathrm{d}x, \quad (3)$$

$$\Pi^D(d, \theta_d, P, F^P) = 0.5 \int_{0}^{P^{-1}(d)} [d + P(y)]\mathrm{d}y + 0.5 \int_{P^{-1}(d)}^{1} [\theta_d + y + 2c]\mathrm{d}y. \quad (4)$$

3. Basic Results

The first question one might ask is whether an NE exists. An affirmative answer is not automatic because we have restricted strategy sets that exclude mixed strategies. It turns out, however, that the BLG has an equilibrium of a very simple form.

Proposition 1. The BLG has an NE in the piecewise-linear, continuous bid functions graphed in Figure 1. The functions are

$$P(\theta_p) = 2\theta_p/3 - 2c + 1/2, \quad (5)$$

truncated above at $\min\{1, 2c + 1/2\}$ and below at $\max\{0, 2c - 1/6\}$ and

$$D(\theta_d) = 2\theta_d/3 + 2c - 1/6, \quad (6)$$

truncated above at $\min\{1, 7/6 - 2c\}$ and below at $\max\{0, -2c + 1/2\}$.

8. A technical clarification: the inverse image of a point z in the range of a bounded nondecreasing left-continuous function D is either a single point $\{b\}$ or an interval $[a, b]$ (or $(a, b]$). We always set $b = D^{-1}(z)$; that is, the function D^{-1} is defined as the supremum of the inverse image correspondence.

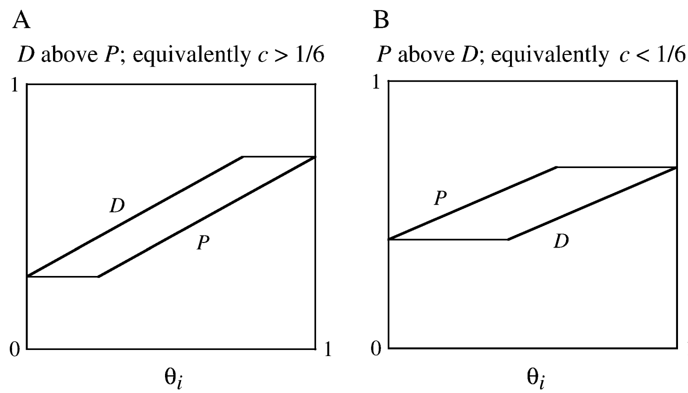

Figure 1. Demand and Offer Curves.

The minimum and maximum values of $d = D(\theta_d)$ are $2c - 1/6$ and $2c + 1/2$ when $\theta_d = 0$ and 1, respectively. So $p = P(\theta_p)$ is never strictly greater than the largest value of D nor strictly below the smallest value. Similarly, d is bounded by the natural range of P.

All proofs appear in the appendices. The intuitive reason for θ_p and θ_d being preceded by a fraction is that each litigant knows that his/her signal is only part of the truth and thus only partially responds to the signal. The intuitive reason for the lower truncation in Figure 1A is that the plaintiff has no incentive to demand less than the lowest possible offer from the defendant. Demanding less would not increase the likelihood of a settlement (it is already 100%) but would reduce what the plaintiff collects in a settlement. Likewise, the upper truncation reflects the fact that the defendant has no incentive to offer more than the plaintiff's highest demand.

Figure 1B shows the perhaps more intuitive case where, at equal signals, the plaintiff demands more than the defendant offers. This case arises when $P(x) = 2x/3 - 2c + 1/2 > D(x) = 2x/3 + 2c - 1/6$. That is, when $c < 1/6$, the plaintiff's demand curve is above the defendant's offer curve. When $c > 1/6$, the plaintiff's demand curve is below the defendant's offer curve as in Figure 1A. The intuitive explanation for the plaintiff demand curve being above the defendant offer curve when the cost of trial is low and below the defendant offer curve when the cost of trial is high is that the plaintiff wants to receive as much as possible and the defendant wants to pay as little as possible. Thus, each side wants to extract the surplus for itself. When court costs are low, the issue of surplus extraction is paramount, and thus, the plaintiff's demand curve is above the defendant's offer curve. As the cost of going to court increases, each side will be more willing to settle. Although the one side's greater willingness to settle increases the intransigence of the other side, this effect is less than the direct effect of the increased cost on the other side. So the net effect of an increase in court cost is a shift downward of the plaintiff's demand curve and a shift upward of the defendant's offer curve.

Note that $(P + D)/2 = \theta_p/3 + \theta_d/3 + 1/6 > (\theta_p + \theta_d)/2$ if and only if the average of the observations is less than 1/2. That is, the distribution of the average of the demand and offer is less extreme than the distribution of the trial outcome were the case to go to trial. Again, this is because the litigants temper their offers relative to the signals.

It is useful to compare these demand and offer curves to those generated by other approaches. First, consider the differential expectations approach. Here there is no point estimate of the demand and offers, only upper and lower bounds, so we can only provide minimal demand and maximal offer curves. For the plaintiff, the minimal demand would be $-c + \theta_p$, and for the defendant, the maximal offer would be $c + \theta_d$. Note that neither litigant takes into account that that the other side may have a different signal. In the one-sided asymmetric information one-side offer case, only one side, say the defendant, has a signal. When the plaintiff makes the demand (as in Bebchuk), then the defendant's offer curve is $c + \theta_d$. The plaintiff does not have a signal in this case, but one could draw a horizontal line to represent the demand by the plaintiff.

One might ask whether there are other NEs. Of course, there are *trivial* NEs, where all cases end in trial because both the plaintiff and the defendant make offers certain to be rejected, for example, $D(\theta) = 1$ and $P(\theta) = 0$ for all signals θ.[9] There are also variations on strategies (5) and (6) that are *inessential* in that they induce the same outcomes (i.e., the same mapping from signals to payoffs). To illustrate, suppose that $c = 1/12$. Then by equation (6) the maximum defendant offer is 2/3, and by equation (5) the plaintiff demand curve is higher and the case will go to trial with probability 1 whenever $\theta_p > 0.5$. Of course, the trial outcome depends only on the signal, not the demand. Hence, the outcome will always be the same if we replace equation (5) by any plaintiff demand function that coincides with equation (5) for $\theta_p \leq 0.5$ and has arbitrary values in $[2/3, \infty)$ for $\theta_p > 0.5$. Thus, some of the truncations in Figure 1 are inessential. However, they keep the graphs of the functions within the unit square, which simplifies later calculations.

Our uniqueness result covers equilibria that are symmetric in the sense that corresponds to the symmetry of the BLG. For example, suppose that when the defendant observes the signal 1/4, he/she offers 1/3. Symmetry would imply that when the plaintiff observes 3/4, he/she demands 2/3. With this in mind, we make the following

Definition. The strategies P and D of the simple litigation game are symmetric if for all θ in [0, 1] we have $P(\theta) = 1 - D(1 - \theta)$ or, equivalently, $D(\theta) = 1 - P(1 - \theta)$.

It is easy to see that the trivial NE strategies mentioned above are symmetric; so are the piecewise-linear strategies (5)–(6). Our next result is that the equilibrium is essentially unique in its class.

Proposition 2. All nontrivial piecewise-linear symmetric NEs of the BLG induce the same outcome as strategies (5)–(6).

In the rest of this section, "equilibrium" refers to the outcome generated by strategies (5)–(6). We focus on the probability of trial and the distribution of

9. We will drop the subscripts from θ_p and θ_d when the meaning is clear.

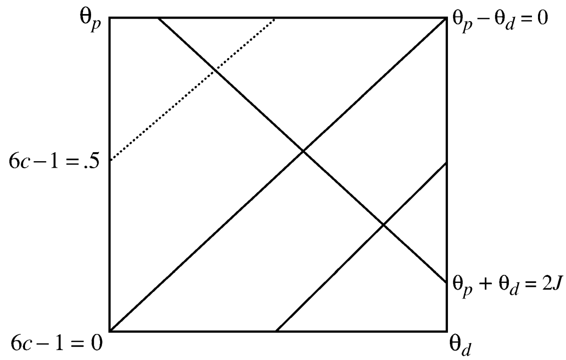

Figure 2. The Probability of a Trial.

cases that go to trial because for empirical work trial data are much easier to collect than data on settlements. The first comparative statics result shows that the probability of a trial decreases nonlinearly in the trial cost c.

Proposition 3. The equilibrium probability of trial is $1 - 18c^2$ for $0 \leq c \leq 1/6$; is $2(1 - 3c)^2$ for $1/6 \leq c \leq 1/3$; and is 0 for $c \geq 1/3$.

Inspection of equations (5)–(6) and Figure 1 provides much of the intuition. A trial is certain if the highest defendant offer ($2/3 + 2c - 1/6$) is below the lowest plaintiff demand ($-2c + 1/2$); that is, if $c \leq 0$. A trial will never occur if the lowest defendant offer ($2c - 1/6$) exceeds (or equals) the highest plaintiff demand ($7/6 - 2c$); that is, if $c \geq 1/3$. For intermediate costs, the case goes to trial when $p = (2/3)\theta_p - 2c + 1/2 > (2/3)\theta_d + 2c - 1/6 = d$, that is, when $\theta_p - \theta_d > 6c - 1$. The last inequality is more likely to be satisfied, and thus, the case is more likely to go to trial, when c is smaller. Although the result that an increase in court cost will increase the probability of a settlement is not surprising, the nonlinear relationship may be. The change in expressions at $c = 1/6$ is associated with the change in Figure 1 from case a (with D above P) to case b (with P above D). Appendix A derives the exact expressions.

Now consider the probability of a trial conditional on the value of the potential trial judgment $J = (\theta_p + \theta_d)/2$. Initial intuition (and the Priest-Klein result) suggests that cases with extreme judgments are more likely to be settled, and cases with judgments near the average of 0.5 are more likely to go to trial. Our next result confirms this intuition for high trial costs, but reverses it for low costs.

Proposition 4. The equilibrium probability of a trial increases in $|J - 0.5|$ when $c < 1/6$ and decreases in $|J - 0.5|$ when $c > 1/6$.

That is, the farther the trial outcome is from the median trial outcome, the more likely that there will be a trial when trial costs are low and the less likely there will be a trial when trial costs are high. The intuition can be extracted from Figure 2. Recall from the discussion of Proposition 3 that a trial will take place if and only if $\theta_p - \theta_d > 6c - 1$, that is, if and only if the signal combination (θ_p, θ_d) lies northwest of the lines of slope +1 labeled $6c - 1 = K$. Consider first the case $K > 0$, that is, $c > 1/6$; say $K = 0.5$ as in Figure 2. The trial region now is the triangle to the northwest of the dotted line. Lines of given value $J = (\theta_p + \theta_d)/2$ have slope -1. For J near zero (and for J near 1), the negatively sloped J line does not intersect the trial triangle. Hence, the trial probability conditioned on such J is zero. For a value of J closer to 0.5, such as that shown in Figure 2, the J line does intersect the trial triangle. Since signals are independent and uniform, their joint distribution is uniform over the unit square. Hence, the desired conditional trial probability is the length of the J line segment inside the triangle as a fraction of its length inside the unit square. This is clearly maximized when the J line meets the corner of the triangle, that is, when $J = 0.5$.

To see the less intuitive case, suppose $c < 1/6$ so $K < 0$ as in the unlabeled upward sloping line below the diagonal in Figure 2. The settlement region now is the triangle southeast of this line, whereas the trial region is the rest of the unit square, northwest of the line. In this case, lines with extreme values of J do not intersect the settlement region, and the trial probability is 1. As J moves toward 0.5, a larger fraction of the J segment lies in the settlement region and the trial probability decreases.

The geometry comes from the fact that when c is small, the plaintiff's demand curve in Figure 1B is above the defendant's offer curve. When the potential judgment J is very high (or very low), the litigants' signals must be fairly similar because J is the average of the signals. When the litigants' signals are similar and c is small, the plaintiff's demand p will tend to be above the defendant's offer d even if θ_p is slightly smaller than θ_d. As a result, the case goes to trial. However, when J is close to 0.5, it is possible that the defendant observed a signal close to 1 and the plaintiff observed a signal close to 0. In such situations, the plaintiff's demand will be below the defendant's offer even though the plaintiff's demand curve is above the defendant's offer curve. That is, there is more scope for signals that will lead to settlement at intermediate levels of the potential judgment. If we interpret θ as probability (and assume that the award is fixed at, say, one million dollars), then this result says that those cases where the plaintiff has a 50% chance of winning are the least likely to go to trial. Thus, the Priest and Klein 50% rule does not hold when the cost of a trial is low.

Of course, when the cost of trial is high, the plaintiff's demand curve is below the defendant's offer curve (as in Figure 1A). As a consequence, small differences that arise when the trial outcome is either close to 0 or 1 will not be sufficient to make the plaintiff's demand above the defendant's offer. So a trial will not take place. On the other hand, when J is near 0.5, it is quite possible for θ_p to be sufficiently larger than θ_d so that p is greater than d, resulting in a trial. Thus, when the cost of a trial is high, our model's results parallel the results of the Priest and Klein model.

Turning our attention to the one-sided asymmetric information one-sided offer model as exemplified by Bebchuk, we get a contrary result. In his model, if the defendant knows the probability of winning and the plaintiff makes the offers, then only when the defendant has a high probability of winning will the defendant reject the plaintiff's offer and the case will go to trial. So that model predicts that trials sample from extreme cases, where the probability of the defendant winning is high.

Our last result for the BLG requires the following definition.

Definition. A random variable x has a *tent distribution* on $[L, U]$ if it has a piecewise-linear continuous density with four pieces: a piece with slope $s > 0$ from L to an intermediate point $M = aU + (1 - a)L$ for some $a \in (0, 0.5)$, a second piece with slope $m \in [0, s]$ from M to $(L + U)/2$, a third piece with slope $-m$ from $(L + U)/2$ to $N = aL + (1 - a)U$, and a final piece from N to U with slope $-s$.

The better-known *triangle distribution* is the degenerate case $m = s$; see Appendix A for further discussion. The uniform distribution is the degenerate case at the other extreme, where $s \to \infty$ and $m = 0$. Hence, tent distributions are unimodal, piecewise-linear distributions between the uniform and the triangular.

Proposition 5. The distribution of *potential* trial judgments is a triangle on [0, 1]. The equilibrium distribution of *actual* trial judgments is a tent on [0, 1] if $0 < c < 1/6$ and is a triangle on $[K/2, (1 - K)/2]$ if $1/6 \leq c < 1/3$, where $K = 6c - 1$.

It is well known that the sum of two independent, uniformly distributed random variables has the triangle distribution, and potential judgments are simply half the sum of the signals. Hence, the first part of the proposition is routine. The equilibrium distribution is less obvious, but the logic from Proposition 4 extends as follows.

- When c is relatively large, the cases that go to trial come from the northwest triangle, and all the extreme cases are settled. Hence, the distribution of observed trial judgments is triangular and concentrated at the center of potential judgments.
- When c is relatively small, cases from the southeast triangle are settled, so the observed trial judgments come from the shaded region of Figure 3. The density height is just a renormalization of the length of the J lines in the trial region. Hence, the distribution in this case is a tent and is more dispersed (closer to uniform) than the overall distribution of potential judgments.

As we have just seen, our model predicts that the larger the cost of trial, the more that trials are concentrated at the center of potential judgments. Assuming that the litigants know the potential amount of the award but not the probability of a verdict in favor of the plaintiff (the assumptions of the Priest-Klein model), this means that an increase in the cost of a trial will result in the

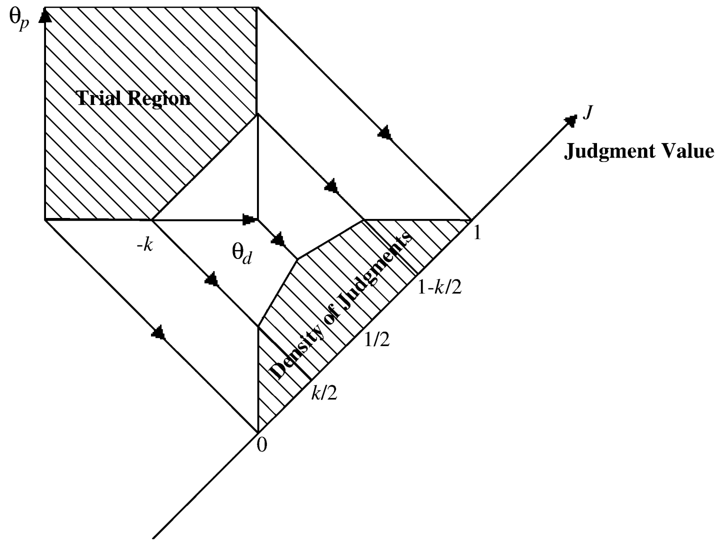

Figure 3. Tent Distribution of Trial Outcomes.

probability of a trial verdict converging toward 50%. Thus, this particular result parallels that of Priest-Klein. This is in contrast to the Bebchuk model, where higher court costs lead to ever more extreme cases being tried. The defendant rejects settlement offers only in those instances where the trial outcome is most likely to be in favor of the defendant. As the cost of going to court increases, the bar is raised for going to trial so that the probability of the defendant winning at trial increases. Waldfogel (1998) shows that the empirical evidence runs contrary to the one-sided asymmetric information one-sided offer approach and in favor of the Priest-Klein approach. Unfortunately, the evidence that Waldfogel provides does not distinguish between our approach and the Priest-Klein approach.

4. Extensions

Additional insights can be gleaned from relaxing the assumptions of the BLG. First, consider shifting the distribution of trial outcomes by adding or subtracting a constant A to the upper and lower end points U and L of the range of possible outcomes. In the litigation game, the probability of trial depends on the differential in expectations, not the level of these expectations. Hence, such shifts in the outcome range have no effect, other things equal. However, thinking carefully about such shifts leads to a more subtle issue concerning the way the litigation game is specified. We shall return to this issue shortly.

Next, consider increasing the width $U - L$ of the trial outcome range, holding constant the cost $c \geq 0$, and maintaining the assumption that signals are independent and uniform on $[L, U]$. We also assume for simplicity (a weaker assumption will do) that L does not increase and refer to such a stretching of possibilities as an *outcome spread*. We have a sharp comparative statics result.

Proposition 6. An outcome spread increases the equilibrium probability of a trial.

Here equilibrium still refers to outcomes generated by the NE strategies (5)–(6), except that the variables are linearly transformed to apply to $[L, U] \neq [0, 1]$. The normalized trial cost c has $U - L$ in the denominator; hence, the relative cost of a trial falls in an outcome spread. In Figure 2, this is represented by a downward (or southeasterly) shift in the K line and, hence, an increase in the area of the trial region (as a fraction of the area of the entire square). Appendix A shows that the desired result turns out to be a corollary of Proposition 3, which tells us that a (relative) cost decrease indeed increases the trial probability.

What happens if we relax the assumption of independent uniformly distributed signals? Signal dependence per se does not appear to introduce any interesting new issues. The signal can be decomposed into a common component that shifts the outcome midrange and idiosyncratic components that are independent. As long as the litigants are risk neutral, it seems that only the idiosyncratic components matter.

The more important generalizations are to independent signals from a distribution that may not be uniform, to trial costs that may not be symmetric, and to bargaining protocols that might not split the difference equally. The ordinal properties of our main propositions survive quite well and extend the intuition of the BLG. There are three parts to the argument.

First, an increase in own trial cost induces more moderate offers and demands. This result is intuitive, but the generality of the setting requires some extra work. Appendix B derives the direct effect $\boldsymbol{d}_c$, the rate at which the defendant's best response offer increases as his/her cost increases, holding constant the plaintiff's demand function. It also derives the indirect effect $\boldsymbol{d}_s$, the rate at which his/her offer increases as the plaintiff's demand shifts up. The plaintiff has an analogous direct effect $\boldsymbol{p}_c$ for an increase in his/her cost and indirect effect $\boldsymbol{p}_s$ for a shift in the defendant's offer function. Appendix B shows that that the direct effects are always toward moderation: $\boldsymbol{d}_c > 0 > \boldsymbol{p}_c$ in the relevant region. The direct effects cause shifts in the best response functions that reverberate via the indirect effects. The total effect of a change in own trial cost is the sum $\boldsymbol{d}_c/(1 - \boldsymbol{d}_s\boldsymbol{p}_s)$ of a geometric series for the defendant. Similarly, $\boldsymbol{p}_c/(1 - \boldsymbol{d}_s\boldsymbol{p}_s)$ is the total effect for the plaintiff. Thus, in the usual case $\boldsymbol{d}_s\boldsymbol{p}_s < 1$, the total effect has the same sign as the direct effect.

In the BLG, the effects turn out to be $\boldsymbol{d}_c = -\boldsymbol{p}_c = 4/3$ and $\boldsymbol{d}_s = \boldsymbol{p}_s = -1/3$. Hence, a unit increase in defendant's cost only will increase his/her offer by $(4/3)/(8/9) = 3/2$ and reduce the plaintiff's demand by $(-1/3)(3/2) = -1/2$. Similarly, a unit increase in plaintiff's cost only will reduce his/her demand by $-3/2$ and increase plaintiff's offer by $1/2$. Thus, we confirm the equilibrium cost coefficients (for equal shifts in both litigants' costs) in equations (5)–(6) of

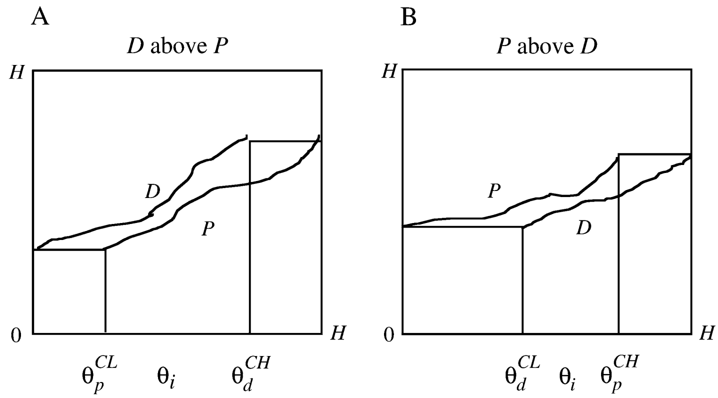

Figure 4. Generalized Demand and Offer Curves.

$3/2 + 1/2 = 2$ and $-1/2 - 3/2 = -2$, and now see that 75% of the impact is due to own cost and 25% due to the other player's cost.

The second part of the argument notes the consequences for extreme signal values. Figure 4 illustrates two key cases. Suppose that in equilibrium D is above P, as in panel A. For reasons given earlier, P will never be below the minimum value of D; hence, $P(\theta_p) = D(L)$ for $\theta_p \leq \theta_p^{CL}$. Similarly, D will never be above the maximum value of P; hence, $D(\theta_d) = P(H)$ for $\theta_d \geq \theta_d^{CH}$. Panel B shows the opposite case where P is above D, and again, each litigant's function has a flat segment over a range of extreme signal values. The same thing happens if the functions were to intersect in the interior. Indeed, there is always a flat segment at both extremes unless (a) both untruncated functions are strictly increasing and (b) $D(L) = P(L)$ and/or $D(H) = P(H)$. But the first part of the argument shows that ties as in (b) are exceptional and will be broken by slightly increasing or decreasing either litigant's trial cost. Hence, we can safely assume a flat segment in one of the litigant's functions at both extremes.

The third and last part of the argument is the same as for Proposition 4, and so is the intuition. When J is either very large or very small, the observations by the litigants must be similar. If the demand curve is below the offer curve, this means that the case will be settled; if the demand curve is above the offer curve, this means that the case will be tried. More specifically, in case A (where D is above P), a settlement is certain for all judgments in the more extreme half of the flat segments ($J < [L + \theta_p^{CL}]/2$ and $J > [H + \theta_d^{CH}]/2$ in panel A, where $L = 0$), and in general, the cases that go to trial oversample the middling judgments.

By the first part of the argument, case A is to be expected when trial costs are high. By the same token, when trial costs are low, we expect to see case B (where P is above D). Now trial cases oversample the more extreme judgments, and the most extreme judgments ($J < [L + \theta_d^{CL}]/2$ and $J > [H + \theta_p^{CH}]/2$ in panel B) are certain to go to trial. Hence, our main qualitative results appear to be quite robust.

To counter a bias we perceive in much of the literature, we have modeled litigation as entirely symmetric. We see no reason to suppose informational or bargaining asymmetries based on who makes the offer, but there is a natural strategic asymmetry: the plaintiff can always drop the case, or not bring it to begin with, whereas the defendant has no such options (unless there is a potential for a countersuit). The general litigation model presented in Section 2 does not allow for this asymmetry, but it may now be worth a brief discussion.

The key question is when the plaintiff would prefer to drop the case after his/her demand has been rejected. A numerical example will help fix ideas. Suppose that $\theta_p = 0$, then $p = 1/2 - 2c$ according to equation (5). If the plaintiff's demand is rejected, he/she can infer $(2/3)\theta_d + 2c - 1/6 < 1/2 - 2c$, that is, $\theta_d < -6c + 1$. If the case goes to trial, he/she expects to get $0.5[0] + 0.5(0.5)[1 - 6c] - c = 0.25 - 2.5c$. Hence $c < 0.1$ implies that the threat of a trial is always credible in the BLG. For $c > 0.1$ in more general models, the threat of a trial is still credible if $c < L$. Assuming $c < L$ is the standard approach used in the literature to ensure that the plaintiff's threat of going to trial is always credible.[10]

5. Concluding Remarks

We obtained a nontrivial NE for the BLG and showed it was essentially unique in its class (Propositions 1–2). In this equilibrium, a trial takes place if and only if $\theta_p - \theta_d > 6c - 1$. It is instructive to compare this result to the traditional divergent expectations story, where a trial takes place if and only if the differential in the litigants' draws are greater than the total cost of going to trial, that is, a trial takes place if and only if $\theta_p - \theta_d > 2c$. In our model the traditional condition is neither necessary nor sufficient. If $c < 1/4$, then $6c - 1 < 2c$ and a trial is possible in equilibrium even though the condition $\theta_p - \theta_d > 2c$ is violated. For example, if $c = 1/6$, then, inconsistent with the traditional divergent expectations story, there will be a trial whenever θ_p is between θ_d and $\theta_d + 1/3$. Similarly, if $c > 1/4$, then $6c - 1 > 2c$ and there will be equilibrium settlements inconsistent with the traditional story. For example, if $c = 1/3$, there will always be a settlement, while the divergent expectations story would predict a trial whenever $\theta_p - \theta_d > 2/3$. In a nutshell, the traditional divergent expectations explanation is misleading. More intuitively, the basic problem with the simple divergent expectations approach is that it does not take into any explicit account one side's expectations about the other side's observations and behavior. In a common-values setting, which characterizes most court cases, each side must take into account that his/her observations are only

10. We do not deal with situations where the plaintiff's expected return from going to trial is negative. In Bebchuk (1986), the plaintiff is informed and the defendant (who makes the offer) is not. Because the defendant can only imperfectly screen, sometimes the defendant makes a positive offer when the net expected value to the plaintiff is negative and therefore the plaintiff would not have taken the case to trial if the defendant had offered nothing. Nalebuff (1987) assumes that the defendant is informed and the plaintiff makes the offers. A rejection of an offer by the defendant may signal to the plaintiff that the case has negative expected value. Incorporating the possibility of a negative expected return from going to trial in the two-sided incomplete information model is an interesting challenge for the future.

part of the picture. The simple divergent expectations approach does not do this. Furthermore, the simple divergent expectations approach does not consider the other side's strategic behavior—it just says that if the signal for the plaintiff minus the signal for the defendant is less than the cost of the trial, then there will be a settlement. And if this is not the case, there will be no settlement. Even if the setting were not common values, each side needs to think strategically. Each side rationally is willing to reduce the probability of settlement in order to extract more surplus from the settlement.

We derived several comparative static results regarding the probability of a trial. We showed that an increase in c (or a decrease in the range of possible outcomes) reduces the probability of a trial (Propositions 3 and 6).

We also showed that when the trial cost is relatively high, trials are more apt to come from the cases with potential judgments near the median (Propositions 4 and 5). To compare this result with that of Priest and Klein, it is again useful to characterize the award as being one million dollars if the plaintiff wins, and to interpret θ_p and θ_d as the plaintiff's and defendant's estimates of the plaintiff's probability of winning. With this interpretation, our result is that cases are less likely to go to trial when the plaintiff's probability of winning is either very low or very high. Hence, our result here is consistent with the argument put forth by Priest and Klein that trials tend to select from cases with a 50% chance of winning.

However, when the trial cost is relatively low, trials are more apt to come from the cases with potential judgments at the extremes (Propositions 4 and 5), contrary to Priest and Klein. Of course, exact comparisons are difficult because their analysis applies to a negligence rule and differs in other respects from our model. We suspect that the primary reason for the contrary results is their use of the traditional divergent expectations approach. Another possible reason is that they implicitly assume an additional error component that increases as the expected outcome approaches the mean outcome. For a further discussion of the Priest and Klein result, see Wittman (1988) and Shavell (1996).

Because information on settlements is often not available, much research is based on trial data. As we have shown, the inferences on settlements drawn from trial data depend greatly on the cost of trials. One needs the models presented here to get a more accurate understanding of all cases—both tried and settled—whenever the data relies only on cases that went to trial. Priest and Klein pointed out the problem of simple extrapolation. Here we have developed a more nuanced model, showing how the litigants make use of signals of common value and how the relative cost of trial can alter the distribution of cases that go to trial.

The work presented here provides several avenues for further research. One suggested by recent events is diplomacy in the shadow of war. Stage 1 of our litigation game can be reinterpreted as diplomatic negotiations between two sovereign nations with conflicting interests. A settlement is possible, but if the demands at stage 1 are incompatible, then there is a costly armed conflict. It seems reasonable to represent the expected outcome of the war by the common-value expression used in the litigation model, $[\theta_p + \theta_d]/2$. To pursue this application, it would make sense to relax the condition $L \geq 0$.[11]

Theorists may enjoy exploring a second general avenue for further research. We suspect that stronger versions of Proposition 2 can be obtained, demonstrating essential uniqueness within broader classes of strategies. One perhaps could characterize precisely the class of distributions and payoff asymmetries, for which the ordinal comparative statics results hold. Appendix B and the discussion in Section 4 is a step in this direction. There are also many variants of the litigation game (e.g., with plaintiff options to not file and to drop after negotiation) that could be formalized and explored.

More important, there is now a new set of ordinal comparative statics results that can be tested indirectly on existing field data or tested directly on laboratory data. It will be particularly interesting to see whether the low–trial cost result can account for observed data, since it contradicts previous intuition and conventional wisdom.

Finally, we believe that the application of the common-values framework to litigation is underexploited relative to the one-sided asymmetric model. We hope that this study will promote further applications in this area.

Appendix A

Lemma 1. Any best response in the litigation game can be represented by a nondecreasing left-continuous function.

Proof. Let D be an arbitrary defendant strategy, and suppose that plaintiff strategy P does not have the desired monotonicity property. We will show that there is a better response, Q, that is closer to monotonic. If P is not nondecreasing, then there are points $a < b$ such that $A = P(a) > P(b) = B$. Let $Q(a) = B$ and $Q(b) = A$ and let $Q(x) = P(x)$ for $x \neq a, b$. The payoff sum at points a and b for Q is half of

$$\Pi_Q = \int_{[d \geq B]} [B + D(y)]\mathrm{d}F(y) + \int_{[d<B]} [a + y - 2c]\mathrm{d}F(y) + \int_{[d \geq A]} [A + D(y)]\mathrm{d}F(y) + \int_{[d<A]} [b + y - 2c]\mathrm{d}F(y).$$

The corresponding sum for P is

$$\Pi_P = \int_{[d \geq A]} [A + D(y)]\mathrm{d}F(y) + \int_{[d<A]} [a + y - 2c]\mathrm{d}F(y) + \int_{[d \geq B]} [B + D(y)]\mathrm{d}F(y) + \int_{[d<B]} [b + y - 2c]\mathrm{d}F(y).$$

11. Of course, one might prefer a less static model of bargaining where war and diplomacy are available every period. But the static model is still useful as it is an upper bound on bargaining efficiency.

The payoff difference is

$$\Pi_Q - \Pi_P = \int_{[d\in[B,A]]} [(b+y-2c)-(a+y-2c)]\mathrm{d}F(y)$$
$$= (b-a)\Pr_F[D(y)\in[B,A)].$$

Since $b > a$ by hypothesis, the last expression is nonnegative, and it is strictly positive if the defendant's offers fall between B and A with positive probability. Hence Q is a better response, and strictly better, except in inessential cases. This establishes the desired nondecreasing property for plaintiff's best response. The argument for the defendant is similar.

Nondecreasing bounded functions on a compact set are well known to have at most a countable set of discontinuity points; hence, left continuity is without loss of generality (see Loeve 1963, 175, for the proof).

The key intuition is that for given demand levels A versus B, the trade-off between trial and settlement favors trial more at the higher signal b because a settlement is directly unaffected by the signal, but a trial outcome increases in the signal. The main idea in the proof is analogous to the "bubble sort" algorithm: whenever one finds a lower demand at a higher signal, one switches the demands, thereby creating a better response. The ultimate effect is that the demand is everywhere (weakly) increasing in the signal, as desired.

Proposition 1. The BLG has an NE in the piecewise-linear, continuous bid functions graphed in Figure 1. The functions are

$$P(\theta_p) = 2\theta_p/3 - 2c + 1/2, \qquad \text{(A1)}$$

truncated above at $\min\{1,\ 2c + 1/2\}$ and below at $\max\{0,\ 2c - 1/6\}$, and

$$D(\theta_d) = 2\theta_d/3 + 2c - 1/6, \qquad \text{(A2)}$$

truncated above at $\min\{1,\ 7/6 - 2c\}$ and below at $\max\{0,\ -2c + 1/2\}$.

Proof. We need only verify that equation (A1) is a best response to equation (A2) and vice versa. Note first that it is impossible for both the demand and offer to equal 1. $P(\theta_p) = 2\theta_p/3 - 2c + 1/2 \geq 1$ only if $c \leq 1/12$. But if $c \leq 1/12$, then $D(\theta_d) = 2\theta_d/3 + 2c - 1/6 \leq 2/3$. A similar exercise shows that both the demand and offer cannot equal 0. Thus, the 0, 1 truncations are inessential because when the truncations are operative, the case would go to trial whether there was a truncation or not, and the demands and offers do not affect the trial outcome. The truncation of $P(\theta_p)$ above at $2c + 1/2$ and the truncation of $D(\theta_d)$ below at $-2c + 1/2$ are also inessential for similar reasons.

The defendant minimizes his/her expected payment

$$\Pi^D(d,\theta_d,P,F^P) = 0.5\int_0^{P^{-1}(d)} [d + P(y)]\mathrm{d}y + 0.5\int_{P^{-1}(d)}^{1} [\theta_d + y + 2c]\mathrm{d}y. \qquad \text{(A3)}$$

Differentiating this expression with respect to d, one obtains the first-order condition:

$$\Pi_d^D = d/P'(P^{-1}(d)) + 0.5P^{-1}(d) \\ - [0.5P^{-1}(d) + 0.5\theta_d + c]/P'(P^{-1}(d)) = 0. \tag{A4}$$

Multiplying equation (A4) through by $P'(P^{-1}(d))$, we get the more convenient expression:

$$d + 0.5P^{-1}(d)P'(P^{-1}(d)) - 0.5\theta_d - c - 0.5P^{-1}(d) = 0. \tag{A5}$$

We first look at the case where $c < 1/6$. In this situation, the plaintiff's demand curve is above the defendant's offer curve and the truncations are inessential; so we can ignore them. We are checking the defendants' best response to $P(\theta_p) = 2\theta_p/3 - 2c + 1/2$, so we substitute $P^{-1}(d) = 3d/2 + 3c - 3/4$ and $P' = 2/3$ in equation (A5) to obtain

$$0 = d - 3d/12 - 3c/6 + 3/24 - 0.5\theta_d - c \\ = 0.75d - 1.5c + 1/8 - 0.5\theta_d. \tag{A6}$$

The unique solution is $d = 2\theta_d/3 + 2c - 1/6$, as desired. Moreover, the second derivative of the objective function is

$$\Pi_{dd}^D = 1.5/P'(P^{-1}(d)) - 0.5/[P'(P^{-1}(d))]^2 \\ = 1.5/(2/3) - 0.5/(4/9) = 9/4 - 9/8 = 9/8 > 0, \tag{A7}$$

so we are indeed at a minimum. Hence, we have verified the best response when $c < 1/6$.

Next suppose that $c \geq 1/6$ so that the defendant offer curve is on or above the plaintiff demand curve and the demand and offer functions involve essential truncations as in Figure 1A. As explained in the text, the truncation does not reduce the probability of a settlement but does make the settlement more favorable for the litigant. The computations above for $c < 1/6$ still hold, but we still need to check the truncations. It suffices to show that even when $\theta_d = 0$, the defendant will still want to settle. Here $d = 2c - 1/6$ and there will be a settlement for that amount. If the defendant were to raise his/her offer, he/she would clearly be worse off. If the defendant were to reduce his/her offer, then there would be a trial. Recall that the plaintiff will demand $2c - 1/6$ for all values of $\theta_p \leq 6c - 1$. Hence, the expected cost of the trial to the defendant would be $0.5(0) + 0.5[0.5(6c - 1)] + c = 2.5c - 0.25$. The term in brackets is the expected value of θ_p given that $\theta_p \leq 6c - 1$. When is the expected cost of a trial, $2.5c - 0.25$, greater than the cost of a settlement, $2c - 1/6$? When $0.5c > 1/12$; equivalently, when $c > 1/6$—the condition for the plaintiff's demand curve to be below the defendant's offer curve.

The game being symmetric, a similar argument verifies that the given piecewise-linear plaintiff strategy is a best response to the defendant's given strategy.

Proposition 2. All nontrivial piecewise-linear symmetric NEs of the BLG induce the same outcome as strategies (5)–(6).

Proof. We will now make use of the symmetry conditions to rewrite equation (A5) as a function of D rather than P. We first explicitly consider the following symmetry relationships.

Let $y = P(z) = 1 - D(1 - z)$. Then $z = P^{-1}(y)$ and $D^{-1}(1 - y) = 1 - z$. Equivalently, $z = 1 - D^{-1}(1 - y)$. So $P^{-1}(y) = 1 - D^{-1}(1 - y)$ and $P'(P^{-1}(d)) = D'(1 - P^{-1}(d)) = D'(1 - 1 + D^{-1}(1 - d)) = D'(D^{-1}(1 - d))$.

Substituting these relationships in equation (A5) we get

$$\begin{aligned} &d + 0.5[1 - D^{-1}(1-d)]D'(D^{-1}(1-d)) - 0.5\theta_d - c - 0.5[1 - D^{-1}(1-d)] \\ &\quad = d + 0.5[1 - D^{-1}(1-d)][D'(D^{-1}(1-d)) - 1] - 0.5\theta_d - c = 0. \end{aligned} \tag{A8}$$

D is assumed to be piecewise linear. Let $d = a_d + b_d c + e_d \theta_d$, then $\theta_d = (d - a_d - b_d c)/e_d$ and $D' = e_d$. Equation (A8) can be rewritten as follows:

$$d + 0.5(e_d - 1) - 0.5(e_d - 1)(1 - d - a_d - b_d c)/e_d - 0.5\theta_d - c = 0. \tag{A9}$$

Equivalently,

$$\begin{aligned} d(1.5e_d - 0.5)/e_d = &-0.5(e_d - 1)(1 - 1/e_d) - 0.5(e_d - 1)a_d/e_d \\ &- 0.5(e_d - 1)(b_d c)/e_d + c + 0.5\theta_d, \end{aligned} \tag{A10}$$

or

$$\begin{aligned} d = &-0.5(e_d - 1)(1 - 1/e_d)e_d/(1.5e_d - 0.5) \\ &- 0.5(e_d - 1)a_d/(1.5e_d - 0.5) - 0.5(e_d - 1)(b_d c)/(1.5e_d - 0.5) \\ &+ ce_d/(1.5e_d - 0.5) + 0.5\theta_d e_d/(1.5e_d - 0.5). \end{aligned} \tag{A11}$$

Now the coefficient of θ_d is e_d. So from equation (A11), we have the following relationship:

$$e_d = 0.5e_d/(1.5e_d - 0.5) = e_d/(3e_d - 1). \tag{A12}$$

The solutions are $e_d = 2/3$, 0. Clearly, the equation pair (A1)–(A2) satisfies these conditions. The inessential truncations need not hold because, as shown in Proposition 1, the probability of a trial is 1 regardless. So the outcome is not changed as long as the slope is greater than or equal to zero once the point of truncation is reached (hence the word "piecewise").

The only question remaining is whether there is another piecewise-linear function with this set of slopes but in a different combination. We first show that $P = 1/2 + 0\theta_p$, $D = 1/2 + 0\theta_d$ is not an NE for all $0 < c < 1/3$. Suppose that

$\theta_p = 1$, then the expected outcome if the case goes to trial is 3/4. So the plaintiff will raise his/her demand above 0.5 if $c < 0.25$.

Every other horizontal line where D is below P always results in trial and therefore is trivial. The reverse can never be a equilibrium as all cases would be settled and the plaintiff would want to increase his/her demand and the defendant would want to decrease his/her offer.

There are two other families of possibilities.

1. The 2/3-slope line is broken up by one or more 0 slope lines.
2. The 2/3-slope line is still in the middle, but the 0 slope line start or stop at a different place.

We will focus on (1). The argument for (2) is a blend of the previous arguments.

Let us consider a horizontal portion between two line segments with 2/3 slope. Moving along the horizontal portion, the defendant's loss from going to trial strictly increases continuously as θ_d increases. Furthermore, we know that that the probability of trial is less than 1 as the defendant increases his/her offer for still larger values of θ_d to reduce the probability of trial. Thus, the defendant should continuously strictly increase his/her offer to continuously reduce the probability of a trial. But a 0 slope says otherwise. Hence, we are led to a contradiction.

Proposition 3. The equilibrium probability of trial is $1 - 18c^2$ for $0 \leq c \leq 1/6$; $2(1 - 3c)^2$ for $1/6 \leq c \leq 1/3$; and 0 for $c \geq 1/3$.

Proof. A case goes to trial when $p = (2/3)\theta_p - 2c + 1/2 > (2/3)\theta_d + 2c - 1/6 = d$, that is, when $\theta_p - \theta_d > 6c - 1$. Clearly, as c increases, the probability of a trial decreases. More specifically, the probability of a trial is 0 if the lowest defendant offer ($2c - 1/6$ in equilibrium) is weakly above the highest plaintiff demand ($7/6 - 2c$ in equilibrium); so in equilibrium the condition reduces to $c \geq 1/3$. In this situation, the plaintiff will demand the minimum defendant offer and the defendant will offer the maximum plaintiff demand. That is, $P(\theta) = D(\theta) = 0.5$ (remember that we are restricting our analysis to nontrivial symmetric equilibria).

Since signals are independent and uniformly distributed, the equilibrium trial probability is the area of the trial region, that is, the subset of signal combinations in the unit square where $\theta_p - \theta_d > 6c - 1$. For $1/6 \leq c \leq 1/3$, the trial region is the isosceles right triangle northwest of the line of slope $+1$ with $\theta_p -$ intercept $6c - 1$. Hence, the triangle has height $h = 1 - (6c - 1) = 2 - 6c$ and area $0.5h^2 = 2(1 - 3c)^2$. For $0 \leq c \leq 1/6$, the trial region excludes only the isosceles right triangle southeast of the line of slope $+1$ with $\theta_d -$ intercept $1 - 6c$. That triangle has height $h = 6c$ and area $0.5h^2 = 18c^2$, and so the trial probability is the remaining area of the unit square, or $1 - 18c^2$.

Proposition 4. The equilibrium probability of a trial increases in $|J - 0.5|$ when $c < 1/6$ and decreases in $|J - 0.5|$ when $c > 1/6$.

Proof. For fixed $J \in [0.5, 1)$, consider the set of signals on the line $(\theta_p + \theta_d)/2 = J$. Since $\theta_p = 2J - \theta_d$ and $J \in [0.5, 1)$, the maximum value of θ_d is 1, in which case $\theta_p = 2J - 1$. Similarly, the maximum value of θ_p is 1, in which case $\theta_d = 2J - 1$. The J line has slope $\Delta\theta_p/\Delta\theta_d$ equal to -1. Hence, the difference $K = \theta_p - \theta_d$ is uniformly distributed over the interval $(2J - 2, 2 - 2J)$.

Recall that a trial only takes place when $\theta_p - \theta_d > 6c - 1$. Hence, the probability $T(J)$ of a trial is the length of the subinterval where $K > 6c - 1$ divided by the length of the interval. The interval is empty and $T(J) = 0$ if $6c - 1 > 2 - 2J$, that is, if $c > 0.5 - J/3$. Otherwise,

$$\begin{aligned} T(J) &= [(2 - 2J) - (6c - 1)]/[(2 - 2J) - (2J - 2)] \\ &= 0.25[3 - 2J - 6c]/[1 - J]. \end{aligned} \tag{A13}$$

To find the effect of an increase in J (the expected trial outcome) on the probability of a trial (T), we take the derivative of T with respect to J over the relevant interval, and simplify to obtain

$$\begin{aligned} T'(J) &= -2(0.25)/[1 - J] + 0.25[3 - 2J - 6c]/[1 - J]^2 \\ &= 0.25[2J - 2 + 3 - 2J - 6c]/(1 - J)^2 \\ &= 0.25[1 - 6c]/[1 - J]^2 \end{aligned} \tag{A14}$$

The sign of T' therefore is the sign of $1 - 6c$. When $c > 1/6$, the probability of a trial decreases, but when $c < 1/6$ the probability of a trial increases as the expected outcome goes from 0.5 to 1. So, the proposition is proven for $J \geq 0.5$. The analysis is similar when $J < 0.5$.

Proposition 5. The distribution of potential trial judgments is a triangle on [0, 1]. The equilibrium distribution of actual trial judgments is a tent on [0, 1] if $0 < c < 1/6$, and is a triangle on $[K/2, (1 - K)/2]$ if $1/6 = c < 1/3$, where $K = 6c - 1$.

Proof. For potential trial judgments, the density is the (integral = 1) normalization of the length $m(J)$ of the constant-J line inside the unit square. Direct computation yields $m(J) = \sqrt{2}\min\{J, 1 - J\}$, whose integral from $J = 0$ to 1 is $1/\sqrt{2}$. Hence, the density is $2\min\{J, 1 - J\}$, which defines a triangular distribution.

For actual trial judgments, recall from proofs of the last two propositions the construction of trial regions in the unit square and the lines of constant signal difference K. First consider $K \geq 0$, that is, $1/6 \leq c < 1/3$, and recall that in this case the trial region is an isosceles triangle northwest of the line with slope $+1$ and y intercept K. The constant-J line through that corner of the triangle has value $J = K/2$. Hence, the trial probability is 0 for $J \leq K/2$. For $J > K/2$, the length of the constant-J segment in the trial region increases linearly in J at rate $\sqrt{2}$ until the line intersects the apex of the trial triangle at $J = 0.5$, as depicted in Figure 3. With further increases in J, the length decreases linearly at the same rate until the line meets the other corner of the trial triangle at $J = (1 - K)/2$. After the integral 1 normalization, we obtain a triangular distribution whose density peaks at

$$2/(1 - K) \quad \text{when}\, J = 0.5.$$

The argument for the case $K < 0$, or $0 < c < 1/6$, is only slightly more complicated and is illustrated in Figure 3. The trial settlement region again is northwest of the line of slope $+1$ whose x intercept is $-K$. The entire J line lies inside the trial region for positive values of $J \leq |K/2|$, and its length (by simple geometry) is $2\sqrt{2}J$. When $J \in (|K/2|, 0.5)$, a portion of the J line falls outside the trial region so its length increases only half as fast, at rate $\sqrt{2}$. Again, for $J > 0.5$ we have a symmetric decrease in segment length inside the trial region. The desired result follows from the proportionality of the density to the segment lengths and the definition of the tent distribution.

Proposition 6. An outcome spread increases the equilibrium probability of a trial.

Proof. Transform a litigation game on a nontrivial interval $[L, U]$ to a BLG on $[0, 1]$ using the mapping $x \rightarrow (x + L)/(U - L)$. It is easy to check that independent uniform signal distributions retain those properties under the transformation. Note that the transformed trial cost is $c = (C + L)/(U - L)$ if the nonnormalized cost is C. A spread is defined as an increase in $U - L = W$, so we consider the partial derivative of $c = (C + L)/W$ with respect to W. It is easily calculated to be $(WL' - C - L)/W^2 \leq -(C + L)/W^2 < 0$, where the weak inequality comes from the proviso that L does not increase in W. Since the normalized cost decreases, we apply Proposition 3 to conclude that the equilibrium trial probability increases.

Appendix B

We analyze the general effect of cost shifts when the defendant has an offer function $\boldsymbol{d} = D(\theta_d)$ and the plaintiff has a demand function $\boldsymbol{p} = P(\theta_p)$. (The notation $\boldsymbol{d}$ distinguishes the offer from the symbol d for a derivative.) We assume that litigants' signals are independent, and the distribution functions F^P and F^D have strictly positive densities f^P and f^D that are differentiable almost everywhere on $[L, H]$. By Lemma 1, we can assume that $P', D' > 0$ and have inverses with positive derivatives in the regions of interest.

Let $Z = P^{-1}(\boldsymbol{d})$. Then $P(Z) = \boldsymbol{d}$ and $Z' = P^{-1'} > 0$. The defendant seeks to minimize

$$2\Pi^D(\boldsymbol{d}, \theta_d, c, P, F^P) \int_L^Z [\boldsymbol{d} + P(y)]\mathrm{d}F^P(y) + \int_Z^H [\theta_d + y + 2c]\mathrm{d}F^P(y). \qquad \text{(B1)}$$

First-order conditions for an interior solution:

$$\begin{aligned}2\Pi^D_d(\boldsymbol{d},\theta_d,c,P,F^P) &= F^P(Z)+[\boldsymbol{d}+P(Z)]f^P(Z)Z'-[\theta_d+Z+2\mathrm{c}]f^P(Z)Z'\\ &= F^P(Z)+2\boldsymbol{d}f^P(Z)\mathrm{Z}'-[\theta_d+Z+2c]f^P(Z)Z'\\ &= F^P(Z)-[\theta_d+Z+2c-2\boldsymbol{d}]f^P(Z)Z'=0. \qquad \text{(B2)}\end{aligned}$$

Since $F^P(Z), f^P(Z)Z' > 0$, equation (B2) implies that

$$[\theta_d+Z+2c-2\boldsymbol{d}]=\frac{F^P(Z)}{f^P(Z)Z'}>0. \qquad \text{(B3)}$$

Second-order conditions (SOCs) for a minimum:

$$\begin{aligned}2\Pi^D_{dd}(\boldsymbol{d},\theta_d,P,F^P) &= 3f^P(Z)Z'-[\theta_d+Z+2c-2\boldsymbol{d}]\\ &\quad\times[f'^P(Z)Z'^2+f^P(Z)Z'']-f^P(Z)Z'^2\\ &= 3f^P(Z)Z'-\frac{F^P(Z)}{f^P(Z)Z'}[f'^P(Z)Z'^2+f^P(Z)Z'']\\ &\quad -f^P(Z)Z'^2>0. \qquad \text{(B4)}\end{aligned}$$

The inequality might not be strict at a set of isolated points, which we can safely ignore.

Equation (B4) is equivalent to

$$\frac{F^P(Z)}{[f^P(Z)Z']^2}[f'^P(Z)Z'^2+f^P(Z)Z'']<3-Z'. \qquad \text{(B5)}$$

Let $\psi=\frac{1}{(\ln(F^P(Z))'}=\frac{F^P(Z)}{f^P(Z)Z'}$. Then by (B3) we have

$$\psi=\theta_d+Z+2c-2\boldsymbol{d}. \qquad \text{(B6)}$$

Using the definition of ψ and equation (B5), we obtain the following relationship:

$$\psi'=1-\frac{F^P(Z)[f'^P(Z)Z'Z'+f^P(Z)Z'']}{[f^P(Z)Z']^2}>Z'-2. \qquad \text{(B7)}$$

In the BLG, $Z''=0=f'^P$ and thus $\psi'=1$.

We now compute the direct effect of an increase in the defendant's cost on the demand, $\boldsymbol{d}_c$, holding constant the plaintiff's demand function, $P(\theta_p)$. Implicitly differentiating equation (B6), we get

$$\psi'\boldsymbol{d}_c=Z'\boldsymbol{d}_c+2-2\boldsymbol{d}_c \qquad \text{(B8)}$$

Thus, the direct effect is

$$\boldsymbol{d}_c=2/(\psi'+2-Z'). \qquad \text{(B9)}$$

Hence, $\boldsymbol{d}_c>0$ if the denominator is greater than 0. But this follows from equation (B7). Thus, the direct effect of an increase in c on D's offer is to increase $\boldsymbol{d}$.

We next compute the effect $\boldsymbol{p}_s$ of a shift in D's offer on P's demand.

Let $W = D^{-1}(\boldsymbol{p})$, then $D(W) = \boldsymbol{p}$, and $W' > 0$ because $D' > 0$. The plaintiff seeks to maximize

$$2\Pi^P(\boldsymbol{p}, \theta_p, c, D, F^D) = \int_W^H [\boldsymbol{p} + D(x)]\mathrm{d}F^D(x) + \int_L^W [\theta_p + x - 2c]\mathrm{d}F^D(x). \quad \text{(B10)}$$

The first-order conditions for an interior solution are then:

$$2\Pi_p^P(\boldsymbol{p}, \theta_p, c, D, F^D) = 1 - F^D(W) + [\theta_p + W - 2c - 2\boldsymbol{p}]f^D(W)W' = 0. \quad \text{(B11)}$$

Since $1 - F^D(W), f^D(W)W' > 0$, equation (B11) implies that

$$[\theta_P + W - 2c - 2\boldsymbol{p}] = \frac{F^D(W) - 1}{f^D(W)W'} < 0. \quad \text{(B12)}$$

SOCs for a maximum:

$$\begin{aligned}2\Pi_{pp}^P(\boldsymbol{p}, \theta_P, c, D, F) &= -3f^D(W)W' + [\theta_p + W - 2c - 2\boldsymbol{p}] \\ &\quad \times [f'^D(W)W'^2 + f^D(W)W''] + f^D(W)W'^2 \\ &= -3f^D(W)W' + \frac{F^D(W) - 1}{f^D(W)W'}[f'^D(W)W'^2 + f^D(W)W''] \\ &\quad + f^D(W)W'^2 < 0. \end{aligned} \quad \text{(B13)}$$

In the BLG this reduces to $-9/2 + 0 + 9/4 < 0$.

Equation (B13) is equivalent to

$$\frac{F^D(W) - 1}{[f^D(W)W']^2}[f'^D(W)W'^2 + f^D(W)W''] < 3 - W'. \quad \text{(B14)}$$

Let $\xi = \dfrac{F^D(W) - 1}{f^D(W)W'}$. Then

$$\xi = \theta_p + W - 2c - 2\boldsymbol{p} \quad \text{(B15)}$$

by equation (B13).

W is the cognate of Z and ξ is the cognate of ψ. That is, the proofs are parallel.

By equation (14),

$$\xi' = 1 - \frac{[F^D(W) - 1][f'^D(W)W'W' + f^D(W)W'']}{[f^D(W)W']^2} > W' - 2. \quad \text{(B16)}$$

Differentiating equation (B15) with respect to c, we obtain

$$\boldsymbol{p}_c = \frac{-2}{\xi' - W' + 2}. \quad \text{(B17)}$$

The denominator is positive by equation (B16); so the expression is negative.

Now consider a shift in the defendant's offer function, D, by a small amount s (in either direction) and derive the sensitivity, $\boldsymbol{p}_s$. At the margin $\boldsymbol{p} = D(\theta_p) + s$, so now set $W = D^{-1}(\boldsymbol{p} - s)$, and the first-order condition is

$$\xi(\boldsymbol{p} - s) = \theta_p + W(\boldsymbol{p} - s) - 2c - 2\boldsymbol{p}. \tag{B18}$$

Holding c constant, we take the derivative of this expression with respect to s and obtain an implicit equation for the desired effect $\boldsymbol{p}_s$.

$$\xi'(\boldsymbol{p} - s)[\boldsymbol{p}_s - 1] + 2\boldsymbol{p}_s = W'(\boldsymbol{p} - s)[\boldsymbol{p}_s - 1]. \tag{B19}$$

In turn, this implies

$$\boldsymbol{p}_s = \frac{\xi' - W'}{\xi' - W' + 2}. \tag{B20}$$

The denominator is positive by equation (B16).

We next compute the indirect effect, $\boldsymbol{d}_s$, of a shift upward in the plaintiff's demand on the defendant's offer. At the margin, $\boldsymbol{d} = P(\theta_d) + s$, so now set $Z = P^{-1}(\boldsymbol{d} - s)$, and the first-order condition is

$$\psi(\boldsymbol{d} - s) = \theta_d + Z(\boldsymbol{d} - s) + 2c - 2\boldsymbol{d}. \tag{B21}$$

Holding c constant, we take the derivative of this expression with respect to s and obtain

$$\psi'(\boldsymbol{d} - s)[\boldsymbol{d}_s - 1] + 2\boldsymbol{d}_s = Z'(\boldsymbol{d} - s)[\boldsymbol{d}_s - 1]. \tag{B22}$$

In turn, this implies

$$\boldsymbol{d}_s = \frac{\psi' - Z'}{\psi' - Z' + 2}. \tag{B23}$$

The total effect of the defendant's cost increase on the defendant's offer includes

1. the direct effect of an increase in cost on the defendant's offer ($\boldsymbol{d}_c$), plus
2. the first-round indirect effect on the defendant's offer: the effect of the change in the defendant's offer on the plaintiff's demand which in turn has an effect on the defendant's offer ($\boldsymbol{d}_c\boldsymbol{p}_s\boldsymbol{d}_s$), plus
3. the second-round indirect effect on the defendant's offer due to the shift at the end of the first round ($\boldsymbol{d}_c[\boldsymbol{p}_s\boldsymbol{d}_s]^2$), etc.

Hence, the total effect is

$$\boldsymbol{d}_c + \boldsymbol{d}_c\boldsymbol{p}_s\boldsymbol{d}_s + \boldsymbol{d}_c[\boldsymbol{p}_s\boldsymbol{d}_s]^2 \cdots = \boldsymbol{d}_c[1 + \boldsymbol{p}_s\boldsymbol{d}_s + [\boldsymbol{p}_s\boldsymbol{d}_s]^2 \cdots] = \frac{\boldsymbol{d}_c}{1 - \boldsymbol{d}_s\boldsymbol{p}_s}. \tag{B24}$$

This expression is positive if $\boldsymbol{p}_s\boldsymbol{d}_s < 1$. Inspection of equations (B20) and (B23) shows that $0 < \boldsymbol{p}_s < 1$ when $\xi' - W' > 0$ and that $0 < \boldsymbol{d}_s < 1$ when $\psi' - Z' > 0$.

In this case, the indirect effect magnifies the direct effect of an increase in c. When either $\xi' - W' < 0$ or $\psi' - Z' < 0$, but not both, then $\boldsymbol{p}_s\boldsymbol{d}_s < 0$, and the indirect mitigates, but does not change, the sign of the direct effect. And if both $\boldsymbol{p}_s$ and $\boldsymbol{d}_s$ are negative but the product is less than 1, then the indirect effect will again magnify the direct effect.

The total effect in equation (B24) has a different sign from the direct effect only in the case (a) $\xi' - W' < 0$, (b) $\psi' - Z' < 0$, and (c) the geometric mean of the absolute indirect effects exceeds 1, so $\boldsymbol{p}_s\boldsymbol{d}_s > 1$. We have not yet been able to find an example.

A similar analysis shows that the total effect of a change in the plaintiff's trial costs is $\boldsymbol{p}_c/(1 - \boldsymbol{d}_s\boldsymbol{p}_s)$. Since the denominator is the same as for defendant's total effect, the same cases determine when the direct effect is magnified, mitigated, or reversed.

In the BLG, $\psi' = \xi' = 1$ and $Z' = W' = 3/2$. So $\boldsymbol{d}_c = 2/(\psi' + 2 - Z') = 4/3$ and $\boldsymbol{d}_s = \psi' - Z'/(\psi' - Z' + 2) = -1/3 = \boldsymbol{p}_s$. Hence, $\boldsymbol{p}_c/(1 - \boldsymbol{d}_s\boldsymbol{p}_s) = (4/3)(9/8) = 3/2$. That is, following a unit increase in his/her own cost, the defendant raises his/her offer (over the relevant range) by 3/2. The impact on the plaintiff is the indirect effect times this total effect, or $(-1/3)(3/2) = -1/2$. By symmetry, if the plaintiff also experiences a unit cost increase, then he/she reduces his/her demand by 3/2, inducing an increase in the defendant's offer by 1/2. Hence, if both experience a unit cost increase, then the total effect is 2, as seen in the equilibrium bid functions.

Technical clarifications

Some readers might find the following remarks useful in reading the mathematical arguments.

- "f truncated above at y" is the function $g(x) = \min\{y, f(x)\}$, and "f truncated below at z" is the function $h(x) = \max\{z, f(x)\}$.
- Lemma 1 establishes that the demand and offer functions can be taken to be weakly increasing everywhere, and they are known to be bounded. Hence by the well-known result from real analysis cited in the text, the functions are differentiable almost everywhere with nonnegative derivative.
- If an SOC holds as a weak inequality everywhere and holds as a strict inequality almost everywhere, then we have a unique interior minimum (or maximum). In this sense, the points where the SOC holds as a weak inequality can be "safely ignored."

References

Ausubel, Lawrence, Peter Crampton, and Raymond J. Deneckere. 2002. "Bargaining with Incomplete Information," in R.J. Aumann and S. Hart, eds., *Handbook of Game Theory*. Vol. 3. Amsterdam, Netherlands: Elsevier Science BV, chap. 50, 1897–945.

Bebchuck, Lucian A. 1984. "Litigation and Settlement under Imperfect Information," 15 *Rand Journal of Economics* 404–15.

———. 1986. "Suing Solely to Extract a Settlement Offer," 17 *Journal of Legal Studies* 437–50.

Chatterjee, Kalyan. 1981. "Comparison of Arbitration Procedures: Models with Complete and Incomplete Information," 11 *IEEE Transactions on Systems, Man and Cybernetics* 101–9.

Chatterjee, Kalyan, and William Samuelson. 1983. "Bargaining under Incomplete Information," 31 *Operations Research* 835–51.

Daughety, Andrew. 2000. "Settlement," in B. Bouckaert and G. De Geest, eds., *Encyclopedia of Law and Economics.* Edgar Elgar.

Daughety, Andrew, and Jennifer Reinganum. 1994. "Settlement Negotiations with Two Sided Asymmetric Information: Model Duality, Information Distribution, and Efficiency," 14 *International Review of Law and Economics* 283–98.

Gould, John. 1973. "The Economics of Legal Conflicts," 2 *Journal of Legal Studies* 279–300.

Kennan, John, and Robert Wilson. 1993. "Bargaining with Private Information," 31 *Journal of Economic Literature* 45–104.

Klemperer, Paul. 1999. "Auction Theory: A Guide to the Literature," 13 *Journal of Economic Surveys* 227–86.

Landes, William. 1971. "An Economic Analysis of the Courts," 14 *Journal of Law and Economics* 61–83.

Loeve, Michel. 1963. *Probability Theory.* 3rd ed. Princeton, N.J.: Van Nostrand.

Nalebuff, Barry. 1987. "Credible Pretrial Negotiation," 18 *Rand Journal of Economics* 198–210.

Priest, George, and Benjamin Klein. 1984. "The Selection of Disputes for Litigation," 13 *Journal of Legal Studies* 1–55.

Schweizer, Urs. 1989. "Litigation and Settlement under Two-Sided Incomplete Information," 56 *Review of Economic Studies* 163–77.

Shavell, Steven. 1982. "Suit, Settlement and Trial: A Theoretical Analysis under Alternative Methods for the Allocation of Legal Costs," 11 *Journal of Legal Studies* 55–81.

———. 1996. "Any Frequency of Plaintiff Victory at Trial Is Possible," 25 *Journal of Legal Studies* 493–501.

Spier, Kathryn. 1992. "The Dynamics of Pretrial Negotiation," 59 *Review of Economic Studies* 93–108.

Waldfogel, Joel. 1998. "Reconciling Asymmetric Information and Divergent Expectations Theories of Litigation," 42 *Journal of Law and Economics* 452–76.

Wittman, Donald. 1988. "Dispute Resolution and the Selection of Cases for Trial: A Study of the Generation of Biased and Unbiased Data," 17 *Journal of Legal Studies* 313–52.