PRODUCERS' MARKETS*
A Model of Oligopoly with Sales Costs

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A producers' market is one in which individual producers set prices and incur most transaction costs, as they engage in sales promotion or marketing activities. I show that non-collusive profit-maximizing behavior leads to the emergence of the lowest marginal-cost producer as the 'price leader' (but not as a monopolist) in a simple producers' market for a homogeneous good. The comparative statics of the simple model (which allows no price discrimination or buyer tradeoff between sales promotion and price) suggest that the prevailing price will not be very responsive to demand shocks, or to cost shocks that do not directly affect the price leader, but will respond via a roughly proportional mark-up to the price leader's costs. On the other hand, marketing costs and profits respond almost proportionately to demand shocks.

1. Introduction

Markets can be usefully distinguished by asking two questions: (a) who sets prices? (b) who handles the logistics of exchange? By a producers' market, I mean a market for which the brief answer to each question is 'the producers of the good'; that is, each producer publicly announces his current price and incurs the main 'selling costs' of putting his good into his customers' hands. Producers' markets are common in modern economies; a small unsystematic sample of goods sold in such markets includes automobiles, bank deposits, and computers. Of course, producers' markets coexist with other market modes, ranging from do-it-yourself search and bilateral negotiation (especially in job and home-ownership markets), through markets dominated by autonomous middlemen (or specialists or brokers), to highly organized impersonal auction markets. The distinction between market modes has clear policy implications: the impact of regulatory or macroecono-

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mic programs depends on market price and quantity responses to cost or demand 'shocks', and these responses depend in turn on the market mode.¹

My purpose in this paper is to analyze a simple non-cooperative model of producers' markets for a homogeneous good. It seems to me that this case best captures the role of selling costs and the struggle for market share. In the very large and unruly class of oligopoly models featuring non-price competition,² the main technical novelties in my model are that price formation is modeled explicitly and differing production costs among producers play an important role. Perhaps the most interesting conclusion to emerge from the analysis is that the lowest-cost producer can be regarded as a price leader: the (non-collusive) equilibrium price is generally his 'preferred price' and typically responds much more sensitively to his cost conditions than to those of his rivals.³

Although the homogeneous good case is technically the simplest and cleanest, it does raise some conceptual issues that should be addressed at the outset. If the selling costs incurred by producers are primarily regarded as a means of reducing the cost to customers of searching for the best price and attribute mix, then these costs would seem to have a negligible role when the good in question is homogeneous and each producer's price is publicly known. Therefore, it is important to recognize that the search process that allows producers and customers to find each other is the first component of exchange logistics. After all, customers do not typically drive to the factory to pick up their packages as they roll off the producer's assembly line. A producer who can deliver the goods and receive payment at a time, place and manner that is convenient to the customer will attract more orders. But this service is costly to provide, since it requires such things as friendly sales representatives, local sales outlets, toll-free telephone lines, lunch invitations, flexible credit terms, etc. In general, customers would have some trade-off between delivered price and such convenience services, but at least for some

¹See Okun (1981) for an extensive discussion of this point. Clower and Leijonhufvud (1975) forcefully argue the inadequacy of the Walrasian model (in which my questions (a) and (b) are answered by a fictitious non-economically motivated 'auctioneer') for addressing these issues. Clower and Friedman (1986) examine the case of markets dominated by inventory-carrying middlemen.

²Two 'classical' references do require mention. Chamberlin (1962, pg. 46 ff) in his 'small numbers case' first defined the problem I investigate. He argued that cartel pricing would arise once 'mutual dependence' – the response of fellow oligopolists to one's own actions – was recognized by each oligopolist. However, his argument contains the implicit assumption that market shares are fixed. My analysis shows that subsequent non-price competition for market shares will typically yield a prevailing price lower than such a cartel price. My model of that non-price competition in section 2 is related to that of Schmalensee (1976), but my comparative statics results are quite different since I do not assume that prevailing price is independent of cost and demand parameters.

³I believe this conception of price leadership corresponds to conversational use among many economists. An earlier version of this paper, Friedman (1984), briefly discusses some of the price leadership literature.
homogeneous goods it seems reasonable to suppose that customers first look for lowest price, and then look for the most convenient producer offering that lowest price. If the good in question represents a large fraction of the customer’s expenditure (and search costs are small due to publicly announced prices and homogeneity), this behavior seems even more reasonable. I have in mind the markets for many industrial products such as cold-rolled steel, portland cement and computer chips, as well as some consumer products such as insured bank deposits.

These practical considerations are intended to justify my assumption that only producers offering their homogenous product at the prevailing (lowest) price obtain a positive market share, and the actual shares of such producers depend on relative selling costs. A few theorists may prefer another justification of my assumptions: this set of assumptions demonstrates that sticky, asymmetric price responses can arise even in the absence of search costs or collusion.

Roughly speaking, the model works as follows. Given efficient price searching by customers, competition will induce all producers to charge the same price. For any given prevailing price, producers will incur sales costs up to the point that increased net revenue due to increased market share equals the increase in sales costs. At equilibrium, in which all incumbent producers operate at this margin of sales costs, each producer’s profits are a determinate function of the prevailing price. In general, the prevailing price that is most profitable for a given producer (his ‘preferred price’) is an increasing function of his marginal production cost. My specification of price competition leads to the result that the prevailing price will generally be the lowest preferred price among producers, and the corresponding producer is identified as the ‘price leader’. Typically, this producer does not monopolize the market as in a simple Bertrand equilibrium, but rather finds it more profitable to maintain the largest market share among several rivals. In comparative statics exercises, equilibrium price responds (roughly) proportio-

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4 In technical terms, the supposition is that customers have lexicographic preferences in cash and ‘convenience’. A purchasing agent of an intermediate good (e.g., a hired employee of a construction firm who chooses the supplier of, say, Portland cement) may have such preferences induced by his employer, and a consumer’s preferences will be almost lexicographic if feasible ‘convenience’ is relatively minor. The empirical observation that different producers of essentially identical goods often offer identical list prices is consistent with this supposition.

5 A referee has kindly pointed out that the good could as well represent a small fraction of a large buyer’s expenditure, e.g., General Motors’ purchase of pencils. He also suggested that the model might apply to a ‘symmetrically differentiated’ product.

6 I have no wish to deny the importance of search costs. One source of inspiration for my approach was Okun’s ‘customer markets’ (1981, Chapter 5) in which customers (particularly for retail goods) form long-term attachments to individual sellers precisely to reduce search costs arising from price dispersion and/or product differentiation. The fact that I can obtain price responses similar to those of Okun, even in a model that has essentially assumed away search costs, suggests that such responses may be more general than previously recognized.
nately to the price leader’s costs but not to those of his rivals, and is quite insensitive to demand shocks.

In the next section I lay out my assumptions and derive preliminary results on the optimal level of selling costs for given configurations of announced prices. I present my equilibrium concept, solve and derive comparative statics results in the following section. The final section discusses the results, generalizations, and directions for further research. Proofs are collected in an appendix.

A few words on notation may be in order. Vectors will typically be indicated by unsubscripted Roman letters, e.g. \( x = (x_1, \ldots, x_n) \), although occasionally underscores will be employed when confusion might otherwise arise. Summations will often be denoted by the subscript \( T \) (for ‘total’), e.g., \( x_T := \sum_{t=1}^n x_t \). The symbol \( := \) means ‘is defined as’. A subscripted \( \partial \) indicates a partial derivative; e.g., \( \partial f / \partial x_1 \). I take the liberty of writing \( f(x_i; x_{-i}) \) for \( f(x_1, \ldots, x_n) \) when I wish to focus on \( x_i \) as opposed to the other arguments \( x_{-i} := (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) \). When it makes for easier reading, I will omit the argument of a function altogether.

2. Marketing equilibrium

I make the following assumptions:

1. There are several (not a ‘large’ number of) producers, each of whom seeks non-collusively to maximize profits; producers are indexed \( i = 1, \ldots, N \), where \( N \geq 2 \).
2. Each producer \( i \) produces the same homogeneous good at some constant positive marginal cost, \( c_i \). There are no capacity constraints, adjustment costs or fixed costs. Indexes are chosen so \( c_1 \leq c_2 \leq \cdots \leq c_N \).
3. The good is sold in a ‘producers’ market’: each producer \( i \) picks his own price \( p_i \) and marketing tactics. The latter are summarized by his sales costs \( s_i \).
4. Customers buy only at the lowest price \( p_0 := \min_j p_j \). Demand is summarized by non-negative smooth function \( Q(p_0) \) with \( Q' < 0 \) whenever \( Q(p_0) > 0 \), and \( R(\infty) = \lim_{p_0 \to \infty} p_0 Q(p_0) < \infty \). The list of producers is chosen so that \( Q(c_N) > 0 \).
5. Market shares \( S_i \) are proportional to sales cost among producers charging \( p_0 \). That is, \( S_i = s_i / s_T \) if \( p_i = p_0 \) and 0 otherwise, where \( s_T \) is the sum of sales costs \( s_j \) over all producers \( j \) such that \( p_j = p_0 \).

Before proceeding with the analysis, a few comments are in order. The first assumption of non-collusive profit maximization should be regarded merely as a standard ‘null hypothesis’ allowing one to employ neoclassical techniques; it will be given more specific content later. \( N \geq 2 \) means (given the last part of Assumption 4) that there are at least two firms whose marginal
costs are not so high as to choke off all demand; there is no presumption that all producers are active. Active producers (i.e., those that sell positive quantities of the good) will be referred to as incumbents.

The second assumption of constant production cost for each firm is a severe simplification, made for analytic tractability. I will discuss the effects of relaxing this assumption in the final section.

Assumption 3 rules out price discrimination, in that $p_i$ is a scalar. For consistency, marketing tactics here exclude secret or personalized price discounts. The assumption of no price discrimination (common in the literature) can be justified by pointing to industry practices such as 'most favored nation clauses' or to technological obstacles for price discrimination. The circumstances under which producers chose price will be discussed in section 3. A full specification of marketing tactics would involve an analysis of the optimal allocation of resources across various activities such as advertising, deployment of sales representatives, arranging convenient delivery schedules, etc. Inasmuch as I wish to focus on other issues, I presume in Assumption 3 that the firm has solved this allocation problem and treat the minimum-cost solution $s_i$ as a decision variable.

Given negligible search costs and the absence of capacity constraints that could result in shortages, Assumption 4 appears to be an adequate characterization of market demand for a non-Giffen good. The assumption that total expenditures on the good converge to a finite value $R(\infty)$ as price goes to $\infty$ is included for technical reasons and is not very restrictive; indeed, one often assumes a finite choke price so $R(\infty) = 0$.

The basic rationale for Assumption 5 has been presented in the introduction. That market shares are directly proportional to sales costs is another analytic simplification. There could well be diminishing returns (or possibly increasing returns) in providing 'convenience' to customers, but the implicit assumption here of constant returns (to relative effort) seems a good starting place. The effects of relaxing this last assumption will also be discussed in the final section.

Inasmuch as a producer's profit depends on others' choices of prices and sales efforts as well as his own, the model has a game-theoretic structure. I have in mind a world in which producers, attempting to increase profits, continually juggle their posted prices and marketing tactics, with all prices immediately observed (and responded to) by each producer, and with marketing tactics unobserved but perhaps eventually partially inferred from market share and other data. Direct analysis of such a continuous-time game of incomplete information seems hopelessly intractable, but an 'approximation' of its steady state(s) in terms of the non-cooperative (i.e. Nash) equilibria of a stylized static game is feasible and in the neoclassical tradition.

In this spirit I pose the producer's decision as a two-stage nested maximization problem as follows.
(a) The marketing problem: given \( p=(p_1, \ldots, p_N) \) and \( s_-(s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_N) \), choose \( s_i \geq 0 \) to maximize

\[
\pi_i = (p_0 - c_i)S_i(s_i, s_-)Q(p_0) - s_i.
\]

Denote a solution by \( s^*_i \), and a simultaneous solution for \( i=1, \ldots, N \) (called a marketing equilibrium, or ME) by \( s^{**}(p)=(s_1^{**}(p), \ldots, s_N^{**}(p)) \).

(b) The pricing problem: given a marketing equilibrium \( s^{**}(p) \) and corresponding profits \( \pi^*_i(p) \) for every \( p \), choose a pricing strategy that maximizes \( \pi^*_i(p) \), given other producers' strategies. Denote the simultaneous solution by \( p^*=(p_1^*, \ldots, p_N^*) \).

I will focus on the marketing problem for the rest of this section. With its solution in hand, it will be easier to address in the next section some subtleties arising in connection with the pricing problem. Thus I take as given for the rest of this section some arbitrary configuration of positive prices \( p=(p_1, \ldots, p_N) \), and call \( p_0 := \min_j p_j \) the prevailing price. Producers seek levels of sales costs \( s_i \) that will maximize profit, which is expressed in (a) above as the product of net unit revenue \((p_0 - c_i)\), market share \( S_i \) and demand \( Q(p_0) \), less sales cost.

First note that any producer \( i \) with \( p_i > p_0 \) maximizes \( \pi_i \) at 0 by setting \( s_i = 0 \), and will in any case obtain \( S_i = 0 \) by Assumption 5. Thus such producers have no effect on other producers' shares or profits and hence can be ignored in the marketing problem. As a matter of notational convenience I will assume that \( p_1 = p_2 = \cdots = p_N = p_0 \) in the remaining paragraphs preceding Proposition 1.

The Kuhn–Tucker conditions for producer \( i \)'s marketing problem (which Proposition 1 will show to be necessary and sufficient under present assumptions) then are

\[
\left\{ \begin{array}{c}
(p_0 - c_i)Q(p_0) \partial_i S_i(s_i, s_-) - 1 \leq 0 \\
\partial_i S_i(s_i, s_-) = \frac{1}{(p_0 - c_i)Q(p_0)} = w_i.
\end{array} \right\}
\]

Thus for each \( i=1, \ldots, n \) either \( s_i^* = 0 \) or \( s_i^* \) satisfies

\[
\partial_i S_i(s_i, s_-) = \frac{1}{(p_0 - c_i)Q(p_0)} = w_i.
\]

The solution \( s^*_i \) (for an individual producer) to the marketing problem is depicted in fig. 1(a). Note that \( w_i \) is shifted upwards (and \( s^*_i \) is reduced) by increases in producer \( i \)'s cost or decreases in demand \( Q(p_0) \), while \( \partial_i S_i \) is shifted inward (and \( s^*_i \) reduced) by increases in sales costs \( s_j \) of rival
D. Friedman, Producers' markets

Fig. 1a. \( w_i \) is inverse gross margin, i.e., \( w_i^{-1} = Q(p_0) (p_0 - c_i) \), which is independent of the sales cost \( s_i \), but \( \partial S_i / \partial s_i \) is of the form \( a(a + s_i)^{-1} \) where \( a = \sum_{j \neq i} s_j \geq 0 \), and so is a declining function of \( s_i \). The text shows that the optimal sales cost \( s_i^* \) typically satisfies \( w_i = \partial S_i / \partial s_i \).

Fig. 1b. MCS is the marginal cost of sales, i.e., the increase in \( s_j \) required to increase market share \( S_0 \) sufficiently to sell a marginal unit when the market demand quantity is \( Q(p_0) \). The discussion in the text shows that \( MCS^{-1} = Q(p_0) \partial S_0 / \partial s_j \). The optimal quantity sold, \( q_i^* \), corresponds to the optimal market share, i.e., \( q_i^* = Q(p_0) S_0(s_i^*, s_{-i}) \). Finally, note that other producers' sales costs, \( s_{-i} \), shift the MCS schedule and hence displace \( q_i^* \) and \( s_i^* \).

producers \( j \). If the curves intersect, the first-order condition (2) can also be depicted as a \( MC = MR \) condition where marginal cost is marginal production cost plus the marginal sales cost required to sell output \( q_i \), and marginal revenue is just \( p_0 \); see the Dean Diagram, fig. 1(b). From either diagram, one can see that \( s_i^* = 0 \) if \( c_i \) or \( a = \sum_{j \neq i} s_j \) is sufficiently high, and/or \( p_0 \) or \( Q(p_0) \) is sufficiently low.

To find the marketing equilibrium, we need a simultaneous solution of eq. (1) for \( i = 1, \ldots, n \). Direct computation yields \( \partial S_i(s_i, s_{-i}) = (s_T - s_i) s_T^{-1} \), so (2) can be rewritten as

\[
s_T - s_i = s_T^2 w_i.
\]

By summing (3) over all \( i \), one can see that a simultaneous solution to (2) must satisfy \( n s_T - s_T = s_T^2 w_T \), so \( s_T = (n-1)/w_T \). Solving (3) for \( s_i \) one obtains

\[
s_i = s_T - s_T^2 w_i (1 - s_T w_i),
\]

and substituting for \( s_T \) we obtain

\[\text{Method of Solution (1) for } s_i.\]

\[\text{Method of Solution (2) for } s_T.\]

\[\text{Method of Solution (3) for } s_i.\]
$$s_i = \frac{n-1}{w_r} \left( 1 - \frac{(n-1)w_i}{w_r} \right).$$  \hspace{1cm} (4)

For sufficiently large \(p_0\), the \(s_i\)'s defined in (4) will be non-negative for all \(i\) and will constitute the unique marketing equilibrium. In general, however, the highest cost producers will not find it profitable to operate (i.e., \(s_i^\ast = 0\) for \(i\) larger than some \(n\)), so the analysis is a bit more complicated. Specifically, let \(n(p_0)\) be the largest \(k \leq N\) for which the following inequality holds:

$$\sum_{j=1}^{k} w_j \geq (k-1)w_k.$$  \hspace{1cm} (5)

Also, let \(p_i^\ast\) denote the lowest \(p_0\) for which (5) holds.\(^8\) The following Proposition justifies the interpretation of \(n(p_0)\) as the number of incumbent producers and \(p_i^\ast\) as producer \(k\)'s entry price.

**Proposition 1** For any \(p = (p_1, \ldots, p_N)\) there exists a unique marketing equilibrium \(s^{\ast\ast}(p)\). The associated market shares \(S_i^\ast := S_i(s^{\ast\ast}(p))\) and profits \(\pi_i^\ast := (p_0 - c_i)S_i^\ast Q(p_0) - s_i^{\ast\ast}\) satisfy

\[
0 = S_i^\ast = s_i^{\ast\ast} = \pi_i^\ast \quad \text{if} \quad p_i > p_0 \quad \text{or} \quad p_0 \leq p_i^\ast \quad \text{and}
\]

\[
\begin{align*}
S_i^\ast &= 1 - (n-1)v_j/v_T \\
s_i^{\ast\ast} &= (n-1)S_i^\ast Q(p_0)/v_T \quad \text{if} \quad p_i = p_0 > p_i^\ast, \\
\pi_i^\ast &= (S_i^\ast)^2(p_0 - c_i)Q(p_0)
\end{align*}
\]

where \(v_T\) is the sum of \(v_j := 1/(p_0 - c_j)\) over all producers \(j\) such that \(\min_k p_k := p_0 = p_j > p_j^\ast\) and \(n\) is the number of such producers.

See the appendix for a proof. Inspection of (6) reveals the following interesting

**Corollary 1** Sales expenditures, market shares and profits among producers posting price \(p_0\) are ordered inversely as marginal production costs at marketing equilibrium. That is, for \(c_1 \leq c_2 \leq \cdots \leq c_n\), we have \(s_1^{\ast\ast} \geq s_2^{\ast\ast} \geq \cdots \geq S_N^\ast\) and similarly for \(S_i^\ast\) and \(\pi_i^\ast\).

Given any set of producers posting the minimum price \(p_0\), one can regard \(\pi_i^\ast\) as a function of \(p_0\) for each such producer \(i\). The price \(p_i^\ast\) that maximizes

\(^8\)It is convenient to make the convention that \(p_i^\ast = c_i\) rather than \(p_i^\ast = 0\); clearly no producer will remain in business if \(p_0 < c_i = \min c_i\).
\( \pi^*_i(p_0) \) is called the\(^9\) preferred price for producer \( i \). The following Proposition will play a pivotal role in the analysis of the pricing problem.

**Proposition 2** Preferred prices \( p^*_i \) exist for each \( i \), and satisfy \( p^*_1 \leq p^*_2 \leq \cdots \leq p^*_n \leq \infty \). Furthermore, \( p^*_i > p^*_i \) and \( \pi^*_i \) is maximized on \([0, p^*_i]\) at \( p^*_i < \infty \) for each \( i \).

The proof, although elementary, is rather tedious and is relegated to the Appendix. A key step is to show that

\[
\pi^*_i(p_0) = QS_i \left\{ 1 + \frac{E}{p_0v_i} + 2 \frac{v_i - v_i^{T2}/v_T}{v_T(n-1) - v_i} \right\}
\]

holds for \( p_0 > p^*_i \) (except at the 'kinks' in \( \pi^*_i \) at entry prices \( p^*_j, \ j > i \)). In this formula, \( E \) is the elasticity of demand at \( Q := Q(p_0) \) (so \( E \) is negative by Assumption 4), and \( v_i^{T2} \) and \( v_T \) are rational functions of \( p_0 \) (with coefficients depending on the \( c_j \)'s) that are independent of \( i \). Starting from the observation that the expression in braces \( \{ \} \) depends **positively** on \( v_i \) (and hence on \( c_i \)), one can show a solution to the first-order condition \( 0 = \pi^*_i(p_0) \), yielding \( p^*_i \), will occur at smaller values of \( p_0 \) for lower \( v_i \), i.e., for producers with lower marginal costs \( c_i \).

The economic intuition behind this result is that in considering reductions in \( p_0 \), a producer trades off profit reductions due to a decreased net unit revenue against profit increases due to enhanced demand, lesser sales costs and possibly greater market share. The balance between these influences on profits is struck at a lower price for lower cost producers, principally because a low cost producer’s net unit revenue is larger – relative to that of his rivals – at lower prices, and so he can better afford promotional activities that enable him to increase market share. It is this last ‘market share effect’ that is captured in the final term in (7).

3. **Equilibrium in prices**

The most direct approach to the pricing problem would be a simple Bertrand specification: each producer’s strategy choice is a price \( p_i \in [0, \infty] \). Non-cooperative equilibrium in this case is \( p_i^* = c_2 \) (or \( c_2 - c \) and \( p_i^* \geq p_i^* \) for \( i \neq 1 \); that is, the low cost producer monopolizes the market at the highest entry-deterring price, and obtains the (possibly quite small) unit profit \( c_2 - c_1 \). Although this specification might be interesting in some contexts (e.g., price-wars), it implicitly assumes that each producer believes he can undercut

\(^9\) If \( \pi^*_i \) is maximized at more than one point, \( p^*_i \) is defined to be the lowest. Generically, the maximum is unique.
his rivals (i.e., pick \( p_i < \min_{j \neq i} p_j \)) without provoking retaliation. Such a belief seems quite naive given our presupposition that producers can immediately observe and respond to each others' prices.\(^\text{10}\) It seems intuitively clear in the present context that a producer \( i \) will not force the prevailing price \( p_0 \) below his preferred price \( p^*_i \), and consequently the lowest preferred price will prevail.

There are several ways one might formalize this intuition.\(^\text{11}\) Perhaps the one requiring the fewest new considerations is to specify a producer \( i \)'s pricing strategy as a choice \((m_i, l_i)\) in \([0, \infty) \times [0, \infty)\), and to determine \( p_i \) by the rule \( p_i = \max\{m_i, p_0\} \) where \( p_0 = \min\{l_1, \ldots, l_N\} \); that is, 'I'll match the prevailing price down to \( m_i \), and undercut the prevailing price down to \( l_i \).' The choice \( l_i \) must satisfy \( l_i \leq m_i \) and is made after every \( m_j \) is known.\(^\text{12}\)

Non-cooperative equilibrium (NE) for the pricing (sub-) game is a selection \((m^*, l^*) = (m^*_1, \ldots, m^*_N, l^*_1, \ldots, l^*_N)\) such that if \( p \) is determined by the rule above, then \( \pi^*_i(p) \) is maximized given \((m^*_{-i}, l^*_{-i})\) by the choice \((m^*_i, l^*_i)\), for each \( i = 1, \ldots, N \). If each \((m^*_i, l^*_i)\) actually maximizes \( \pi^*_i(p) \) whether or not \((m_{-i}, l_{-i}) = (m^*_{-i}, l^*_{-i})\), i.e., is a (weakly) dominant strategy, then we have a Full equilibrium (FE) of the game.

An interpretation of the pricing game in terms of fictitious institution may clarify ideas. In a ‘Dutch Sellers’ (DS) auction,\(^\text{13}\) the pointer on a ‘price clock’ moves steadily downward from a (sufficiently high) initial value and determines the prevailing price \( p_0 \) and individual sellers’ posted prices \( p_i \) as follows. Each seller observes the clock and partially controls it with a button he can push twice. The first push (corresponding to \( l_i \)) indicates that the seller wishes to see \( p_0 \) fall no further; the pointer stops when each seller has pushed his button at least once (or at zero, if some seller never pushes his button) and its final reading defines \( p_0 \). A second push of the button (corresponding to \( m_i \)) indicates that seller \( i \) no longer wishes to match the prevailing price, and \( p_i \) is set at the clock’s reading at that point. If a seller \( i \) never pushes his button a second time, then \( p_i = p_0 \).

A moment’s reflection on the DS auction reveals that a producer cannot improve on the strategy of pushing his button for the first time when the

\(^{10}\)See the opening paragraphs of Schmalensee (1976) for the argument that non-price competition is exempt from this criticism; i.e., that the Cournot-like ME concept is adequate. Also recall that I regard \( \xi \) as not directly observable by other producers. The key point, however, is that small changes in \( p_i \) (but not in \( \xi \)) can have large effects on \( \pi_0 \).

\(^{11}\)Among these are Quick Response Equilibrium of Anderson (1984), the ‘convolutions’ of Marschak and Selten (1978), and the Consistent Conjectures Equilibrium of Bresnahan (1981). See Friedman (1984, pp. 27–29) for a brief justification of the present approach. It should be stressed that each of these approaches include my FE \( \pi^*_i \) among its equilibria.

\(^{12}\)In formal terms, then, the producers actually play a three-stage extensive form game, with non-negative plays \( m_i \) and \( l_i \) in the stages pricing 1, pricing 2 and marketing. The equilibrium concept is subgame perfect NE.

\(^{13}\)See Cassaday (1967, p. 193 ff) for a description of somewhat similar buyers’ auctions.
clock pointer has declined to his preferred price \( p_t^* \), and not pushing it a
second time\(^{14} \) until after it has declined to his entry price \( p_t^e \). Thus the
following Proposition should come as no surprise. A proof appears in the
appendix.

**Proposition 3** If \((m^*, l^*)\) is a NE, then the prevailing price \( p_0 \) lies in the
interval \([c_1, p_1^*]\), and posted prices \( p_i \) coincide with \( p_0 \) for each \( i \) such that
\( p_0 > p_t^i \). If \((m^*, l^*)\) is actually an FE, then \( p_0 = p_1^* \). The choice \((m_1, l_1) = (0, p_1^*)\)
for \( i = 1, \ldots, N \) is a FE.

Note that this Proposition establishes existence of FE (and hence NE) by
explicit example, and establishes the uniqueness of FE prices. In conjunction
with the previous Propositions, then, one can obtain explicit characterizations
of FE sales costs, market shares and profits in terms of production cost
and demand parameters.

There are several reasons one should regard the FE outcome as more
significant than other NE outcomes. First, of course, is the presumption that
if a simple (weakly) dominant strategy is available, it will be selected over a
strategy that is only sometimes a 'best response'. Second, one observes that
the FE outcome yields at least as high profits (and strictly higher profits for
incumbents) as any other NE outcome, i.e., it is Pareto-superior. Finally,
other outcomes seem relatively fragile. For example, one can sustain
\( p_0 \in [c_1, c_2] \) only if non-incumbents (who post \( p_i = p_0 \) but set \( s_i = 0 \)) frighten
the sole incumbent \#1 with the prospect that they will take over the market
if he raises his price \( p_1 \) above \( p_0 \). The threat is empty: such non-incumbents
would obtain negative profits in that event, and one would expect that \( p_0 \)
would soon adjust upward. Such fragility is present (albeit in attenuated
form) as long as \( p_0 \leq p_t^i < p_1^* \) for any \( i \neq 1 \).

Therefore I shall regard the FE outcome as the prediction of the model. In
particular, the equilibrium prevailing price \( p_0 \) is the lowest preferred price
among incumbent producers, viz. the lowest cost producer's preferred price,
\( p_1^* \). However, one is really justified in calling this producer 'the price leader'
only the extent that his preferred price depends much more on his own
circumstances than on those of his rivals. A quick examination of the model's
comparative statics, i.e., the sensitivity of \( p_0 = p_1^* \) to the cost and demand
parameters, is therefore in order.

Recall that \( p_1^* \) is typically a solution of the first-order condition \( \pi_1^*(p_0) = 0 \).
In this case, one can explicitly obtain \( dp_1^*/dc \) by implicitly differentiating the

\(^{14}\) Recall from the discussion preceding Proposition 2 that \( p_1^* \) and \( p_t^i \) generally depend on the
set of producers that have not exited from the auction. Hence these strategies require that
'second pushes' be public knowledge. On the other hand, it that 'second pushes' are
not required in equilibrium. In the statement of Proposition 3, \( p_1^* \) is defined relative to the full set
of producers.
bracketed expression in (7), but the results are rather messy. It is clear from
inspection of (7), however, that small increases in the price leader’s cost \( c_i \)
increase the equilibrium price directly through the middle term\(^{15}\)
\( E(p_0 - c_i)/p_0 \), as well as through the \( v_1 \) and \(-v_1\) appearing respectively in the
numerator and denominator of the last (‘market share’) term. On the other
hand, small changes in \( c_i, i \neq 1 \), exert only an indirect effect on price through
the expressions \(-v_1^{(2)}/v_T\) and \( v_T/(n - 1) \) in the market share term. Inasmuch
as the sign of this indirect effect is ambiguous, and the last two expressions
are relatively insensitive to \( c_i \), my characterization of producer \#1 as the
price leader seems justified.\(^{16}\)

Two qualifications are in order: (1) if (as may sometimes be the case) the
price leader’s preferred price is equal to the entry price for some ‘marginal’
producer \( j \), then small changes in \( c_j \) rather than in \( c_1 \) will dominate changes
in equilibrium price, according to the model. (2) Cost changes large enough
to affect entry or exit decisions, or to yield a new price leader, obviously
have a more complex effect on equilibrium price.

Finally, note that \( Q \) doesn’t enter into the computation of \( p_1^* \), so
equilibrium price is insensitive to (elasticity-preserving) demand shifts. Of
course, by formula (6), sales expenditures and profits will respond proportionately
to demand shifts.

4. Discussion

I can summarize my argument as follows. For a homogeneous good not
subject to price discrimination produced by heterogeneous firms (differing in
production costs) that market their own output, one can expect efficient price
searching by consumers to lead all producers to announce the same
‘prevailing’ price, with sales promotion being the most evident form of
competitive activity. In non-collusive equilibrium, the prevailing price is
governed by the ‘price leader’, a producer whose production costs lead him
to desire the lowest prevailing price. In the simple case analyzed in this
paper, one can unambiguously identify the price leader as the lowest-cost
producer, and derive comparative static results.

What does the formal analysis have to say about the behavior of actual

\(^{15}\)Indeed, one can show that this effect by itself would lead to a proportional mark-up rule, as
for a simple monopolist. Combining this with the direct effects arising from the market share
term, one can conclude that a small percentage increase in \( c_1 \) generally leads to a slightly larger
percentage increase in equilibrium price.

\(^{16}\)A simple numerical example may be in order. For a tripoly facing demand \( Q(P_0) =
100P_0^{1.5} \) with very similar marginal costs \( c_i \) of 1.00, 1.05 and 1.10, a numerical simulation
indicates an FE price \( P_1^* = 2.31 \) (for comparison, the other preferred prices \( P_2^* = 3.10, P_3^* = 3.90 \)
are near or above the cartel price). A 0.01 cost increase for \#1, the ‘price leader’, raises the FE
price by about 0.15, while the same cost increase for either of the other producers lowers the FE
price by about 0.05. When more producers are active and costs are more widely dispersed, one
would expect a greater asymmetry.
markets, say the market for computer chips? The comparative statics suggest that if the low-cost producer (NEC, say) finds a way to decrease unit cost slightly, NEC will lower its prices more than proportionately and obtain a larger market share at lower sales costs for everyone. On the other hand, if a higher cost producer (TI, say) obtains a similar cost reduction, TI optimally will not try to undercut the prevailing price, but will rather increase promotional activities and obtain a larger market share. (TI's rivals will in equilibrium also increase promotional activities, but to a lesser degree.) Although it does not appear explicitly in our (FE) equilibrium formulas, there is an asymmetry in the price mechanism when it comes to NEC's cost increases: rivals are under less immediate pressure to match its price increases (although it is still in their interest to do so) because the old FE prevailing price remains a NE after the cost increase. Hence prices may actually be more 'sticky' upwards (in the sense of less frequent but larger price changes) than downwards!

At this juncture it is worth asking how sensitive the model's rather sharp predictions are to modifications of the assumptions. In Friedman (1984, section 3), I relax the assumptions of constant production cost and proportional market shares, referred to as the 'standard case' assumptions in that paper. Given suitable generalizations of the concepts employed here (e.g., one must specify what happens to demand if there are 'shortages', which can arise given capacity constraints), the basic results obtained here remain valid, with the major exceptions that the lowest marginal cost producer is not necessarily the price leader (i.e., the producer with lowest preferred price), and the comparative statics become more complex — e.g., demand shifts can affect the FE price given capacity constraints. However, price responses certainly remain sticky and asymmetric.

More fundamentally, what happens if one allows some heterogeneity among the producers' goods, or buyer trade-offs between price and convenience, or price discrimination by producers? Anderson (1984), employing an independently conceived but related model, finds that his results go through as long as the profit functions corresponding to my \( \pi^*_T(p) \) obey a certain condition that would be satisfied if (for instance) goods differed according to fixed quality differentials. Of course reasonably complete answers to these questions require a model that derives demand for each producer from underlying preferences. Some preliminary work indicates that the main results survive in a spatial model with a fixed structure of transportation costs, and also as a limiting case in more general preference models. However, much work remains to be done to gain a theoretical understanding of more general producers' markets.

17Of course, dynamic considerations may also be important for an industry in which cost innovations are frequent and anticipated. The remarks in this paragraph are intended largely to suggest empirical applications and tests of the simple model.
To return to the opening theme, one could ask when a producer's market structure will arise, rather than some more nearly 'Walrasian' structure, such as one dominated by independent brokers. It should be noted that total sales costs are not closely related to intrinsic transactions costs in the present model, but respond rather sensitively and erratically to the distribution of production costs. In particular, there will be circumstances in which sales costs are mostly 'rent-seeking' by producers, and consumers (and perhaps even some producers) would be better off under an alternative market structure. The general question of which circumstances lead to the existence or relative efficiency of a given market mode is pivotal and remains wide open.

Appendix: Proofs

Lemma 1. Inequality (5) holds for all $k \leq n(p_0)$. The function $n(p_0)$ is piecewise constant and non-decreasing, and $n(p_0) = N$ for $p_0 > p^*_k$. The entry prices satisfy $c_k \leq p^*_k \leq c_k + (k-1)(c_k - c_1)$ for all $k = 1, \ldots, N$; in particular $p^*_k = c_k$ for $k = 1, 2$.

Proof. Note that $0 \leq w_1 \leq w_2 \leq \cdots \leq w_n$ if $Q(p_0) > 0$ and $c_1 \leq c_2 \leq \cdots c_n \leq p_0$, so if (5) fails for some $k < N$, we have

$$\sum_{j=1}^{k+1} w_j = \sum_{j=1}^{k} w_j + w_{k+1} < (k-1)w_k + w_{k+1} \leq [(k+1)-1]w_{k+1},$$

and (5) fails for $k + 1$. Thus (5) holds for all $k \leq n(p_0)$ and (by definition) fails for all $k > n(p_0)$.

The continuity of $w_i$ in $p_0 > c_i$ follows from the continuity of demand $Q$, and yields continuity of $n(p_0)$ except at the entry prices $p^*_k$. Since $n(p_0)$ is integer-valued, it is piecewise constant. Since the RHS of (5) decreases more rapidly in $p_0$ than the LHS where the inequality holds, it follows that $n(p_0)$ is non-decreasing. Since $w_i w_i \to 1$ as $p_0 \to \infty$, (5) holds for all $p_0$ sufficiently large; that is (by definition) for $p_0 \geq p^*_k$. Note that (5) holds trivially for $k = 1, 2$, and $p_0 \geq c_2$, so $p^*_2 = c_2$ (by convention) and $p^*_k = c_k$. Finally, observe that $kw_1 \leq \sum_{j=1}^{k} w_j$ so $kw_1 \geq (k-1)w_k$ is sufficient for (5). But, from the definitions of $w_1$ and $w_k$, this last inequality is equivalent to $k(p_0 - c_k) \geq (k-1)(p_0 - c_1)$, or $p_0 \geq (k-1)(c_k - c_1) + c_k$. Thus $p^*_k \geq (k-1)(c_k - c_1) + c_k$. An analogous argument yields $p^*_k \geq (k-1)(c_k - c_{k-1}) + c_k$. Q.E.D.

Proof of Proposition 1. From Lemma 1, one can readily verify that $S^*_T$ as defined in (6) is positive, and straightforward computations show that it is equal to $s^{**}_T / s^{**}_T$, and that $s^{**}_T = (n-1)Q(p_0)/v_T$. Then it is easy to check that (2) holds if $p_i = p_0 > p^*_i$. Otherwise, one uses inequality (5) to conclude that
\( \hat{c} \cdot S_{i}(0,s^*;i) = s^*_{i} \cdot (s^*_{i})^2 = \nu_{i}^2 \cdot ((n-1)Q(p_{i})) < w_{i} \) if \( p_{i} = p_{0} \leq p_{i}^{*} \), and of course \( \hat{c} \cdot S_{i}(0,s^*;i) = 0 < w_{i} \) if \( p_{i} > p_{0} \); in either case, the Kuhn-Tucker condition (1) is satisfied for \( s_{i} = 0 \). Hence the specified values for \( s^*(p) \) represent a simultaneous solution for (1) which must be unique by the argument preceding eq. (4). That \( s^*(p) \) is indeed a marketing equilibrium will follow if the expression in (6) represents a profit maximum (rather than a minimum), since the Kuhn-Tucker solution is unique, and profits become negative for \( s_{i} \) sufficiently large (e.g., \( s_{i} > p_{0}Q(p_{0}) \) certainly implies \( \pi_{i} < 0 \)) but are zero for \( s_{i} = 0 \).

Since \( \pi_{i}^* \) as specified in (6) is clearly positive (recall \( p_{i} > p_{i}^* \geq c_{i} \) here), one must only verify the formula to complete the proof. In the following computation, \( \nu_{i}/Q(p_{0}) \).

\[
\begin{align*}
\pi_{i}^*(p_{0}) := & (p_{0} - c_{i})Q(p_{0})S_{i}(s^*;i) - s^*_{i}^2 \\
= & w_{i}^{\frac{-1}{2}}s^*_{i}/s^*_{i} - s^*_{i}^2 = s^*_{i}^2 \left( \frac{1}{w_{i}^2} - 1 \right) \\
= & \frac{n - 1}{w_{i}^{n-1}} \left( \frac{1}{w_{i}^{n-1}} - 1 \right) \\
= & \left( 1 - \frac{n - 1}{w_{i}^{n-1}} \right) \frac{1}{w_{i}} \left( \frac{1}{w_{i}^{n-1}} - 1 \right) \\
= & (S_{i}^*)^2(p - c_{i})Q(p_{0}).
\end{align*}
\]

Proof of Proposition 2. By perhaps deleting some producers from the original list, we may assume that \( p_{i} = p_{0} \) for \( i = 1, \ldots, N \), that \( 0 < c_{1} \leq c_{2} \leq \cdots \leq c_{N} \), and that \( p_{i}^{*} < p_{0} \), where \( p_{0} = \sup \{p_{0}; Q(p_{0}) > 0 \} \) is the 'choke price', possibly \( \infty \). Thus for \( \nu_{i} := (p - c_{i})^{-1} \) we have \( 0 < \nu_{i} \leq \nu_{2} \leq \cdots \leq \nu_{k} \) if \( p_{0} \geq p_{k}^{*} \geq c_{k} \). Now \( n := \max \{k; p_{0} \geq p_{k}^{*} \} \) and \( \nu_{T} := \sum_{i=1}^{n} \nu_{i} \) are both positive and piecewise continuous in \( p_{0} \) with jump discontinuities at \( p_{0} = p_{j}^{*} \), \( j \geq 3 \), by Lemma 1. However, the quotient \((n-1)/\nu_{T} \) is continuous, since (by inequality (5) of the text) both discontinuities are fractional increases of \((j-2)^{-1}\). Hence all the functions specified in Proposition 1 are continuous, and are differentiable except perhaps for \( p_{0} \in \{p_{1}^{*}, p_{2}^{*}, \ldots, p_{N}^{*} \} := D_{i} \).

For \( p_{0} \notin D_{i} \), we may compute, abbreviating \( S_{i}^* \) as \( S_{i} \), \( Q(p_{0}) \) as \( Q \) and \( P_{0}Q/Q \) as \( E \),

\[
\pi_{i}(p_{0}) = QS_{i}^{2} + (1/\nu_{i})QS_{i}^{2} + (2/\nu_{i})QS_{i}S_{i} \\
= QS_{i}^{2}\{1 + E/(p_{0}v_{i}) + 2S_{i}/(v_{i}S_{i})\}.
\]
Now,

\[
S_i = -(n-1) \left[ \frac{v_i}{v_T} \right] = -(n-1) \left[ \frac{v_i - \frac{\alpha_{j}(\eta^2)}{v_T}}{v_T} \right]
\]

\[
= \frac{(n-1)}{v_T} \left[ -v_i^2 - \frac{\alpha_{j}(\eta^2)}{v_T} \right]
\]

\[
= \frac{n-1}{v_T} \left[ v_i - \frac{\alpha_{j}(\eta^2)}{v_T} \right].
\]

where \(\alpha_{j}(\eta^2) = \sum_{j=1}^{h} v_j^2\). Substituting this expression for \(S_i\) and \(S_i = 1 - (n-1)v_i/v_T\) into \{ \} and assuming \(n > 1\), one obtains eq. (7):

\[
\pi_i(p_0) = Q S_i^2 \left\{ 1 + \frac{E}{p_0 v_i} + 2 \frac{v_i - \alpha_{j}(\eta^2)/v_T}{v_T/(n-1)-v_i} \right\}.
\]

To establish existence of \(p_i^*\), observe that as \(p_0 \to \infty\) we have \(Q(p_0) \to R(\infty) \to 0^\infty\) (by Assumption 5), \(v_i/v_T \to 1/N\), and \(S_i \to 1-(N-1)/N = 1/N\), so \(\pi_i^* \to R(\infty)/N^2\). Since \(\pi_i\) is bounded and continuous on \([0, \infty]\), it must be maximized at some \(p_i^*\). Formula (6) allows us to conclude that \(\pi_i(p_0) \geq 0\) if \(Q(p_0) > 0\) and \(p_0 > p_i^*\); hence \(p^* < p_i^* \leq p' \leq \infty\). In the case \(R(\infty) = 0\) (e.g., \(p^* < \infty\)) we obviously have \(p_i^* \to \infty\) for all \(i\). However, if \(R(\infty) > 0\), then \(E(\infty) = -1\) and the sign of \(\pi_i^*\) is determined by the sign of \(S_i^*\) for large \(p_0\) (see the equation for \(\pi_i^*\) preceding (7)). Since \(S_i^*\) at least is always negative, we may conclude that \(\pi_i^* < 0\) for \(p_0 < \infty\) sufficiently large, so \(p_i^* < \infty\).

There remains only to show that the \(p_i^*\)'s have the proper ordering and are maximized on \([0, p_i^*]\) at \(p_i^*\). Given the properties of \(\pi_i\) already derived, \(p_i^*\) will obviously be either \(\infty\); a critical point (i.e., a solution to \(\pi_i(p_0) = 0\)), or a point in \(D = \{p_1^*, \ldots, p_n^*\}\) (in which case \(\lim_{h \to 0} \pi_i(p_i^* + h^2) \geq \lim_{h \to 0} \pi_i(p_i^* - h^2)\)). If \(p_i^*\) is the first (i.e., smallest \(p_0\) relative maximum on \([p_i^*, p']\)), then it is easy to see from (7) that \(\pi_i > 0\) on \([p_i^*, p']\) for \(j \geq i\), so \(p_i^* \geq \tilde{p}_i^*\), and the proof is complete in this case. However, if there are several relative maxima for \(\pi_i\), and the global maximum does not occur at the first, the following Lemma suffices.

**Lemma 2** Suppose \(p_1\) and \(p_2\) are relative maxima of \(\pi_i^*\), and \(\pi_i^*(p_1) \leq \pi_i^*(p_2)\).

If \(k \geq j\), then \(\pi_i^*\) is all maximized on \([p_1, p_2]\) at \(p_2\).

**Proof.** To simplify notation, let \(\pi_i\) continue to denote \(\pi_i^*\), \(S_i\) denote \(S_i^*\), etc. Since \(\pi_i\) is continuous and piecewise differentiable, \(\pi_i\) is the integral of its
derivative for each $i$, and it suffices to show $\int_{p_1}^{p_2} \pi_i(p) \, dp \geq 0$ for all $p \in [p_1, p_2]$. Now for such $p$,

$$\int_{p_1}^{p_2} \pi_i(p) \, dp_0 = \int_{p_1}^{p_2} \pi_i(p) \, dp_0 + \int_{p_1}^{p_2} \pi_i'(p) \, dp_0$$

$$\geq \int_{p_1}^{p_2} \pi_i(p) - \pi_i'(p) \, dp_0$$  \hfill (a)

$$\geq \int_{p_1}^{p_2} Q S_i^2 \{ E/p_0(1/v_k - 1/v_j) + 2(v_k - t)/(\tilde{v} - v_k) - 2(v_j - t)/(\tilde{v} - v_j) \} \, dp_0 \geq 0.$$  \hfill (b)

The justification for (a) is that $\pi_j$ is by hypothesis maximized at $p_2$ so the integral of $\pi_j'$ must be non-negative. For (b), I first employed $0 \leq S_k \leq S_j$ from Proposition 1; $S_k$ refers to $S_k$ if $v_j < t$ and to $S_j$ in the opposite case. Then I used $E < 0$ and $v_k \geq v_j$ to conclude that the first term in \{ \} is non-negative, and the inequalities

$$\frac{v_k - t}{\tilde{v}} \leq \frac{v_k - t}{\tilde{v}} \leq \frac{v_j - t}{\tilde{v}}$$

obtain a non-negative second term. \textit{Q.E.D.}

**Proof of Proposition 3.** First note that $l_i(m) = l_i(m_1, \ldots, m_N)$ in general; that is, $l_i$ is picked in full knowledge of which producers will match a price $p_0$ (and higher prices). Thus the set of (potential incumbent) producers required to define $p^*_i$ and $p^*_i$ can be deduced from $m$; we will employ the notation $p^*_i(m)$ and $p^*_i(m)$ where appropriate in this proof to make this dependence explicit.

The first part of the Proposition is proved by contradiction. Assume that $(m^*, l^*(m))$ is a NE and $p_0 := \min(l_{1-1}^*(m^*), \ldots, l_N^*(m^*)) > p_i^*(m^*)$. Then from the definition of $p^*$, producer $\neq 1$ could increase profits by reducing $l_1^*(m^*)$ to $p^*_i$, contradicting the hypothesis on $p_0$. The supposition that $p_i^*(m^*) > p_i^*(0) := p_i^*$ also leads to a contradiction: in this case there must be some $i$ who could earn positive profits at the prevailing price of $p^*_i$ but has set $m_i^* > p_i^*$ and guaranteed $\pi_i = 0$, so $m_i^*$ is not a best response. Finally, if $p_0 < c_1$, then the demand $Q(p_0)$ is positive and satisfied by a producer (or producers) whose unit cost exceeds prices. Such producers obtain $\pi_i < 0$, which is impossible in NE since setting $m_i = l_i$ sufficiently high can guarantee $\pi_i = 0$. Hence
$p_0 \in [c_i, p^*_i]$ in NE. That $p_i = p_0$ if $p_0 > p^*_i(m)$ in any best response is immediate from Proposition 1, and consequently $p^*_i(m) = p^*_i(0) = p^*_i$.

The rest of the Proposition can be proved directly. Since the best i can do if $p_0 \in (0, p^*_i)$ is obtain $n_i = 0$, producer i will be indifferent among choices of $m_i$ in $[0, p^*_i]$. However, a choice of $m_i > p^*_i$ will yield $n_i = 0$ rather than $n_i > 0$ whenever $(m_{i-1}, l_{i-1})$ yields $p_0 \in (p^*_i, m_i)$. Hence $m^*_i \in [0, p^*_i]$ is (weakly) dominant, and $l^*(m) = p^*_i(m)$ is also dominant by definition of preferred prices and Proposition 2. Therefore $(m, l) = (0, p^*)$ is a FE, and for any FE$(l^*, m^*)$,

$p_0 := \min l^*(m) = \min_p p^*_i(m^*) = p^*_i(m^*) = p^*_i(0) = p^*_i$. Q.E.D.

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