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PRODUCERS' MARKETS*

A Model of Oligopoly with Sales Costs

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A producers' market is one in which individual producers set prices and incur most transaction costs, as they engage in sales promotion or marketing activities. I show that non-collusive profit-maximizing behavior leads to the emergence of the lowest marginal–cost producer as the 'price leader' (but not as a monopolist) in a simple producers' market for a homogeneous good. The comparative statics of the simple model (which allows no price discrimination or buyer tradeoff between sales promotion and price) suggest that the prevailing price will not be very responsive to demand shocks, or to cost shocks that do not directly affect the price leader, but will respond via a roughly proportional mark-up to the price leader's costs. On the other hand, marketing costs and profits respond almost proportionately to demand shocks.

1. Introduction

Markets can be usefully distinguished by asking two questions: (*a*) who sets prices? (*b*) who handles the logistics of exchange? By a *producers' market*, I mean a market for which the brief answer to each question is 'the producers of the good'; that is, each producer publicly announces his current price and incurs the main 'selling costs' of putting his good into his customers' hands. Producers' markets are common in modern economies; a small unsystematic sample of goods sold in such markets includes automobiles, bank deposits, and computers. Of course, producers' markets coexist with other market modes, ranging from do-it-yourself search and bilateral negotiation (especially in job and home-ownership markets), through markets dominated by autonomous middlemen (or specialists or brokers), to highly organized impersonal auction markets. The distinction between market modes has clear policy implications: the impact of regulatory or macroecono-

*The work presented here is an offshoot of joint research with Robert Clower, indeed, it was his insistence on the importance of sales costs in certain sorts of market structures that led to the present model. I am also indebted to Robert Anderson, Frederick Balderston, Michael Hagerty, Mark Isaac, Thomas Marschak, Stanley Reynolds and participants in UCLA and Berkeley seminars for useful discussions. An anonymous referee provided an exceptionally thoughtful set of comments on conceptional issues and guidance for a final revision. Aryeh Friedman provided helpful research assistance.

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mic programs depends on market price and quantity responses to cost or demand 'shocks', and these responses depend in turn on the market mode.[1]

My purpose in this paper is to analyze a simple *non-cooperative* model of producers' markets for a *homogeneous* good. It seems to me that this case best captures the role of selling costs and the struggle for market share. In the very large and unruly class of oligopoly models featuring non-price competition,[2] the main technical novelties in my model are that price formation is modeled explicitly and differing production costs among producers play an important role. Perhaps the most interesting conclusion to emerge from the analysis is that the lowest-cost producer can be regarded as a *price leader*: the (non-collusive) equilibrium price is generally his 'preferred price' and typically responds much more sensitively to his cost conditions than to those of his rivals.[3]

Although the homogeneous good case is technically the simplest and cleanest, it does raise some conceptual issues that should be addressed at the outset. If the selling costs incurred by producers are primarily regarded as a means of reducing the cost to customers of searching for the best price and attribute mix, then these costs would seem to have a negligible role when the good in question is homogeneous and each producer's price is publicly known. Therefore, it is important to recognize that the search process that allows producers and customers to find each other is but the first component of exchange logistics. After all, customers do not typically drive to the factory to pick up their packages as they roll off the producer's assembly line. A producer who can deliver the goods and receive payment at a time, place and manner that is convenient to the customer will attract more orders. But this service is costly to provide, since it requires such things as friendly sales representatives, local sales outlets, toll-free telephone lines, lunch invitations, flexible credit terms, etc. In general, customers would have some trade-off between delivered price and such convenience services, but at least for some

[1] See Okun (1981) for an extensive discussion of this point. Clower and Leijonhufvud (1975) forcefully argue the inadequacy of the Walrasian model (in which my questions (a) and (b) are answered by a fictitious non-economically motivated 'auctioneer') for addressing these issues. Clower and Friedman (1986) examine the case of markets dominated by inventory-carrying middlemen.

[2] Two 'classical' references do require mention. Chamberlin (1962, pg. 46 ff) in his 'small numbers case' first defined the problem I investigate. He argued that cartel pricing would arise once 'mutual dependence' – the response of fellow oligopolists to one's own actions – was recognized by each oligopolist. However, his argument contains the implicit assumption that market shares are fixed; my analysis shows that subsequent non-price competition for market shares will typically yield a prevailing price lower than such a cartel price. My model of that non-price competition in section 2 is related to that of Schmalensee (1976), but my comparative statics results are quite different since I do *not* assume that prevailing price is independent of cost and demand parameters.

[3] I believe this conception of price leadership corresponds to conversational use among many economists. An earlier version of this paper, Friedman (1984), briefly discusses some of the price leadership literature.

homogeneous goods it seems reasonable to suppose that customers first look for lowest price, and then look for the most convenient producer offering that lowest price.[4] If the good in question represents a large fraction of the customer's expenditure (and search costs are small due to publicly announced prices and homogeneity), this behavior seems even more reasonable.[5] I have in mind the markets for many industrial products such as cold-rolled steel, portland cement and computer chips, as well as some consumer products such as insured bank deposits.

These practical considerations are intended to justify my assumption that only producers offering their homogenous product at the prevailing (lowest) price obtain a positive market share, and the actual shares of such producers depend on relative selling costs. A few theorists may prefer another justification of my assumptions: this set of assumptions demonstrates that sticky, asymmetric price responses can arise even in the absence of search costs[6] or collusion.

Roughly speaking, the model works as follows. Given efficient price searching by customers, competition will induce all producers to charge the same price. For any given prevailing price, producers will incur sales costs up to the point that increased net revenue due to increased market share equals the increase in sales costs. At equilibrium, in which all incumbent producers operate at this margin of sales costs, each producer's profits are a determinate function of the prevailing price. In general, the prevailing price that is most profitable for a given producer (his 'preferred price') is an increasing function of his marginal production cost. My specification of price competition leads to the result that the prevailing price will generally be the *lowest* preferred price among producers, and the corresponding producer is identified as the 'price leader'. Typically, this producer does *not* monopolize the market as in a simple Bertrand equilibrium, but rather finds it more profitable to maintain the largest market share among several rivals. In comparative statics exercises, equilibrium price responds (roughly) proportio-

[4]In technical terms, the supposition is that customers have lexicographic preferences in cash and 'convenience'. A purchasing agent of an intermediate good (e.g., a hired employee of a construction firm who chooses the supplier of, say, Portland cement) may have such preferences induced by his employer, and a consumer's preferences will be almost lexicographic if feasible 'convenience' is relatively minor. The empirical observation that different producers of essentially identical goods often offer identical list prices is consistent with this supposition.

[5]A referee has kindly pointed out that the good could as well represent a small fraction of a large buyer's expenditure, e.g., General Motors' purchase of pencils. He also suggested that the model might apply to a 'symmetrically differentiated' product.

[6]I have no wish to deny the importance of search costs. One source of inspiration for my approach was Okun's 'customer markets' (1981, Chapter 5) in which customers (particularly for retails *goods*) form long-term attachments to individual sellers precisely to reduce search costs arising from price dispersion and/or product differentiation. The fact that I can obtain price responses similar to those of Okun, even in a model that has essentially assumed away search costs, suggests that such responses may be more general than previously recognized.

nately to the price leader's costs but not to those of his rivals, and is quite insensitive to demand shocks.

In the next section I lay out my assumptions and derive preliminary results on the optimal level of selling costs for given configurations of announced prices. I present my equilibrium concept, solve and derive comparative statics results in the following section. The final section discusses the results, generalizations, and directions for further research. Proofs are collected in an appendix.

A few words on notation may be in order. Vectors will typically be indicated by unsubscripted Roman letters, e.g. $x=(x_1,\ldots,x_n)$, although occasionally underscores will be employed when confusion might otherwise arise. Summations will often be denoted by the subscript T (for 'total'), e.g., $x_T:=\sum_{i=1}^n x_i$. The symbol $:=$ means '(is) defined as'. A subscripted ∂ indicates a partial derivative; e.g., $\partial_i f(x_1,\ldots,x_n)=\partial f(x)/\partial x_i$. I take the liberty of writing $f(x_i;x_{-i})$ for $f(x_1,\ldots,x_n)$ when I wish to focus on x_i as opposed to the other arguments $x_{-i}:=(x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_n)$. When it makes for easier reading, I will omit the argument of a function altogether.

2. Marketing equilibrium

I make the following assumptions:

(1) There are several (not a 'large' number of) producers, each of whom seeks non-collusively to maximize profits; producers are indexed $i=1,\ldots,N$, where $N\geqq 2$.

(2) Each producer i produces the same homogeneous good at some constant positive marginal cost, c_i. There are no capacity constraints, adjustment costs or fixed costs. Indexes are chosen so $c_1\leqq c_2\leqq\cdots\leqq c_N$.

(3) The good is sold in a 'producers' market': each producer i picks his own *price* p_i and marketing tactics. The latter are summarized by his *sales costs* s_i.

(4) Customers buy only at the lowest price $p_0:=\min_j p_j$. *Demand* is summarized by non-negative smooth function $Q(p_0)$ with $Q'<0$ whenever $Q(p_0)>0$, and $R(\infty)=\lim_{p_0\to\infty} p_0Q(p_0)<\infty$. The list of producers is chosen so that $Q(c_N)>0$.

(5) Market shares S_i are proportional to sales cost among producers charging p_0. That is, $S_i=s_i/s_T$ if $p_i=p_0$ and 0 otherwise, where s_T is the sum of sales costs s_j over all producers j such that $p_j=p_0$.

Before proceeding with the analysis, a few comments are in order. The first assumption of non-collusive profit maximization should be regarded merely as a standard 'null hypothesis' allowing one to employ neoclassical techniques; it will be given more specific content later. $N\geqq 2$ means (given the last part of Assumption 4) that there are at least two firms whose marginal costs are not so high as to choke off all demand; there is no presumption that all producers are active. Active producers (i.e., those that sell positive quantities of the good) will be referred to as *incumbents*.

The second assumption of constant production cost for each firm is a severe simplification, made for analytic tractability. I will discuss the effects of relaxing this assumption in the final section.

Asssumption 3 rules out price discrimination, in that p_i is a scalar. For consistency, marketing tactics here exclude secret or personalized price discounts. The assumption of no price discrimination (common in the literature) can be justified by pointing to industry practices such as 'most favored nation clauses' or to technological obstacles for price discrimination. The circumstances under which producers chose price will be discussed in section 3. A full specification of marketing tactics would involve an analysis of the optimal allocation of resources across various activities such as advertising, deployment of sales representatives, arranging convenient delivery schedules, etc. Inasmuch as I wish to focus on other issues, I presume in Assumption 3 that the firm has solved this allocation problem and treat the minimum-cost solution s_i as a decision variable.

Given negligible search costs and the absence of capacity constraints that could result in shortages, Assumption 4 appears to be an adequate characterization of market demand for a non-Giffen good. The assumption that total expenditures on the good converge to a finite value $R(\infty)$ as price goes to ∞ is included for technical reasons and is not very restrictive; indeed, one often assumes a finite choke price so $R(\infty)=0$.

The basic rationale for Assumption 5 has been presented in the introduction. That market shares are *directly* proportional to sales costs is another analytic simplification. There could well be diminishing returns (or possibly increasing returns) in providing 'convenience' to customers, but the implicit assumption here of constant returns (to relative effort) seems a good starting place. The effects of relaxing this last assumption will also be discussed in the final section.

Inasmuch as a producer's profit depends on others' choices of prices and sales efforts as well as his own, the model has a game-theoretic structure. I have in mind a world in which producers, attempting to increase profits, continually juggle their posted prices and marketing tactics, with all prices immediately observed (and responded to) by each producer, and with marketing tactics unobserved but perhaps eventually partially inferred from market share and other data. Direct analysis of such a continuous-time game of incomplete information seems hopelessly intractible, but an 'approximation' of its steady state(s) in terms of the non-cooperative (i.e. Nash) equilibria of a stylized static game is feasible and in the neoclassical tradition.

In this spirit I pose the producer's decision as a two-stage nested maximization problem as follows.

(*a*) The *marketing problem*: given $p=(p_1,\dots,p_N)$ and $s_{-i}=(s_1,\dots,s_{i-1},s_{i+1},\dots,s_N)$, choose $s_i \geqq 0$ to maximize

$$\pi_i=(p_0-c_1)S_i(s_i,s_{-i})Q(p_0)-s_i.$$

Denote a solution by s_i^*, and a simultaneous solution for $i=1,\dots,N$ (called a *marketing equilibrium*, or *ME*) by $s^{**}(p)=(s_1^{**}(p),\dots,s_N^{**}(p))$.

(*b*) The *pricing problem*: given a marketing equilibrium $s^{**}(p)$ and corresponding profits $\pi_i^*(p)$ for every p, choose a pricing strategy that maximizes $\pi_i^*(p)$, given other producers' strategies. Denote the simultaneous solution by $p^*=(p_i^*,\dots,p_N^*)$.

I will focus on the marketing problem for the rest of this section. With its solution in hand, it will be easier to address in the next section some subtleties arising in connection with the pricing problem. Thus I take as given for the rest of this section some arbitrary configuration of positive prices $p=(p_1,\dots,p_N)$, and call $p_0:=\min_j p_j$ the *prevailing price*. Producers seek levels of sales costs s_i that will maximize profit, which is expressed in (*a*) above as the product of net unit revenue (p_0-c_i), market share S_i and demand $Q(p_0)$, less sales cost.

First note that any producer i with $p_i>p_0$ maximizes π_i at 0 by setting $s_i=0$, and will in any case obtain $S_i=0$ by Assumption 5. Thus such producers have no effect on other producers' shares or profits and hence can be ignored in the marketing problem. As a matter of notational convenience I will assume that $p_1=p_2=\cdots=p_n=p_0$ in the remaining paragraphs preceding Proposition 1.

The Kuhn–Tucker conditions for producer i's marketing problem (which Proposition 1 will show to be necessary and sufficient under present assumptions) then are

$$\left.\begin{array}{l}(p_0-c_i)Q(p_0)\,\partial_i S_i(s_i,s_{-i})-1\leqq 0\\[1ex] \mathrm{s_i}\geqq 0\end{array}\right\}\text{c.s.}\tag{1}$$

Thus for each $i=1,\dots,n$ either $s_i^*=0$ or s_i^* satisfies

$$\partial_i S_i(s_i,s_{-i})=\frac{1}{(p_0-c_i)Q(p_0)}=:w_i.\tag{2}$$

The solution s_i^* (for an *individual* producer) to the marketing problem is depicted in fig. 1(a). Note that w_i is shifted upwards (and s_i^* is reduced) by increases in producer i's cost or decreases in demand $Q(p_0)$, while $\partial_i S_i$ is shifted inward (and s_i^* reduced) by increases in sales costs s_j of rival

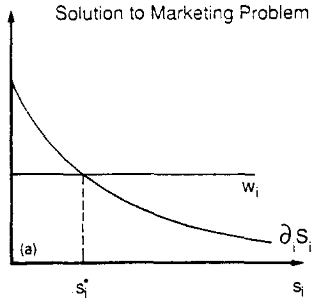

Fig. 1a. w_i is inverse gross margin, i.e., $w_i^{-1} = Q(p_0)(p_0 - c_i)$, which is independent of the sales cost s_i, but $\partial_i S_i(s_i, s_{-i})$ is of the form $a(a+s_i)^{-2}$ where $a = \sum_{j \neq i} s_j \geqq 0$, and so is a declining function of s_i. The text shows that the optimal sales cost s_i^* typically satisfies $w_i = \partial_i S_i$.

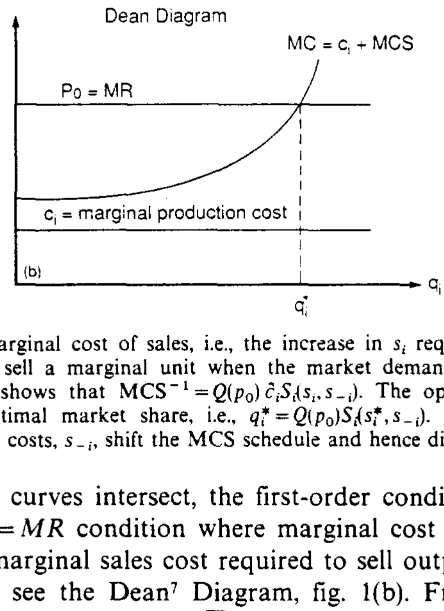

Fig. 1b. MCS is the marginal cost of sales, i.e., the increase in s_i required to increase market share S_i sufficiently to sell a marginal unit when the market demand quantity is $Q(p_0)$. The discussion in the text shows that $\mathrm{MCS}^{-1} = Q(p_0)\,\partial_i S_i(s_i, s_{-i})$. The optimal quantity sold, q_i^*, corresponds to the optimal market share, i.e., $q_i^* = Q(p_0) S_i(s_i^*, s_{-i})$. Finally, note that other producers' sales costs, s_{-i}, shift the MCS schedule and hence displace q_i^* and s_i^*.

producers j. If the curves intersect, the first-order condition (2) can also be depicted as a $MC = MR$ condition where marginal cost is marginal production cost *plus* the marginal sales cost required to sell output q_i, and marginal revenue is just p_0; see the Dean[7] Diagram, fig. 1(b). From either diagram, one can see that $s_i^* = 0$ if c_i or $a = \sum_{j \neq i} s_j$ is sufficiently high, and/or p_0 or $Q(p_0)$ is sufficiently low.

To find the marketing equilibrium, we need a simultaneous solution of eq. (1) for $i = 1, \ldots, n$. Direct computation yields $\partial_i S_i(s_i, s_{-i}) = (s_T - s_i)/s_T^2$, so (2) can be rewritten as

$$s_T - s_i = s_T^2 w_i. \tag{3}$$

By summing (3) over all i, one can see that a simultaneous solution to (2) must satisfy $ns_T - s_T = s_T^2 w_T$, so $s_T = (n-1)/w_T$. Solving (3) for s_i one obtains $s_i = s_T - s_T^2 w_i(1 - s_T w_i)$, and substituting for s_T we obtain

[7] After Joel Dean (1951, p. 356 ff).

$$s_i = \frac{n-1}{w_T}\left(1 - \frac{(n-1)w_i}{w_T}\right). \tag{4}$$

For sufficiently large p_0, the s_i's defined in (4) will be non-negative for all i and will constitute the unique marketing equilibrium. In general, however, the highest cost producers will not find it profitable to operate (i.e., $s_i^* = 0$ for i larger than some n), so the analysis is a bit more complicated. Specifically, let $n(p_0)$ be the largest $k \leqq N$ for which the following inequality holds:

$$\sum_{j=1}^{k} w_j \geqq (k-1)w_k. \tag{5}$$

Also, let p_k^e denote the lowest p_0 for which (5) holds.[8] The following Proposition justifies the interpretation of $n(p_0)$ as the *number of incumbent producers* and p_k^e as producer k's *entry price.*

Proposition 1 For any $p = (p_1, \dots, p_N)$ there exists a unique marketing equilibrium $s^{**}(p)$. The associated market shares $S_i^* := S_i(s^{**}(p))$ and profits $\pi_i^* := (p_0 - c_i)S_i^* Q(p_0) - s_i^{**}$ satisfy

$$0 = S_i^* = s_i^{**} = \pi_i^* \quad \textit{if} \quad p_i > p_0 \textit{ or } p_0 \leqq p_i^e \quad \textit{and}$$

$$\begin{cases} S_i^* = 1 - (n-1)v_i/v_T \\ s_i^{**} = (n-1)S_i^* Q(p_0)/v_T \quad \textit{if} \quad p_i = p_0 > p_i^e, \\ \pi_i^* = (S_i^*)^2 (p_0 - c_i) Q(p_0) \end{cases} \tag{6}$$

where v_T is the sum of $v_j := 1/(p_0 - c_j)$ over all producers j such that $\min_k p_k := p_0 = p_j > p_j^e$ and n is the number of such producers.

See the appendix for a proof. Inspection of (6) reveals the following interesting

Corollary 1 Sales expenditures, market shares and profits among producers posting price p_0 are ordered inversely as marginal production costs at marketing equilibrium. That is, for $c_1 \leqq c_2 \leqq \cdots \leqq c_n$, we have $s_1^{**} \geqq s_2^{**} \geqq \cdots \geqq S_n^{**}$ and similarly for S_i^* and π_i^*.

Given any set of producers posting the minimum price p_0, one can regard π_i^* as a function of p_0 for each such producer i. The price p_i^* that maximizes

[8] It is convenient to make the convention that $p_1^e = c_1$ rather than $p_1^e = 0$; clearly no producer will remain in business if $p_0 < c_1 = \min c_i$.

$\pi_i^*(p_0)$ is called the[9] *preferred price* for producer i. The following Proposition will play a pivotal role in the analysis of the pricing problem.

Proposition 2 Preferred prices p_i^* exist for each i, and satisfy $p_1^* \leqq p_2^* \leqq \cdots \leqq p_N^* \leqq \infty$. Furthermore, $p_i^* > p_i^e$ and π_i^* is maximized on $[0, p_1^*]$ at $p_1^* < \infty$ for each i.

The proof, although elementary, is rather tedious and is relegated to the Appendix. A key step is to show that

$$\pi_i^*(p_0) = QS_i^2 \left\{ 1 + \frac{E}{p_0 v_i} + 2 \frac{v_i - v_T^{(2)}/v_T}{v_T/(n-1) - v_i} \right\} \tag{7}$$

holds for $p_0 > p_i^e$ (except at the 'kinks' in π_i^* at entry prices p_j^e, $j > i$). In this formula, E is the elasticity of demand at $Q := Q(p_0)$ (so E is negative by Assumption 4), and $v_T^{(2)}$ and v_T are rational functions in p_0 (with coefficients depending on the c_j's) that are independent of i. Starting from the observation that the expression in braces { } depends *positively* on v_i (and hence on c_i), one can show a solution to the first-order condition $0 = \pi_i^*(p_0)$, yielding p_i^*, will occur at smaller values of p_0 for lower v_i, i.e. for producers with lower marginal costs c_i.

The economic intuition behind this result is that in considering reductions in p_0, a producer trades off profit reductions due to a decreased net unit revenue against profit increases due to enhanced demand, lesser sales costs and possibly greater market share. The balance between these influences on profits is struck at a lower price for lower cost producers, principally because a low cost producer's net unit revenue is larger – relative to that of his rivals – at lower prices, and so he can better afford promotional activities that enable him to increase market share. It is this last 'market share effect' that is captured in the final term in (7).

3. Equilibrium in prices

The most direct approach to the pricing problem would be a simple Bertrand specification: each producer's strategy choice is a price $p_i \in [0, \infty]$. Non-cooperative equilibrium in this case is $p_1^* = c_2$ (or $c_2 - \varepsilon$) and $p_i^e \geqq p_1^*$ for $i \neq 1$; that is, the low cost producer monopolizes the market at the highest entry-deterring price, and obtains the (possibly quite small) unit profit $c_2 - c_1$. Although this specification might be interesting in some contexts (e.g., price-wars), it implicitly assumes that each producer believes he can undercut

[9] If π_i^* is maximized at more than one point, p_i^* is defined to be the lowest. Generically, the maximum is unique.

his rivals (i.e., pick $p_i < \min_{j \neq i} p_j$) without provoking retaliation. Such a belief seems quite naive given our presupposition that producers can immediately observe and respond to each others' prices.[10] It seems intuitively clear in the present context that a producer i will not force the prevailing price p_0 below his preferred price p_i^*, and consequently the lowest preferred price will prevail.

There are several ways one might formalize this intuition.[11] Perhaps the one requiring the fewest new considerations is to specify a producer i's pricing strategy as a choice (m_i, l_i) in $[0, \infty) \times [0, \infty)$, and to determine p_i by the rule $p_i := \max\{m_i, p_0\}$ where $p_0 = \min\{l_1, \ldots, l_N\}$; that is, 'I'll match the prevailing price down to m_i, and undercut the prevailing price down to l_i.' The choice l_i must satisfy $l_i \geqq m_i$ and is made after every m_j is known.[12] *Non-cooperative equilibrium* (NE) for the pricing (sub-) game is a selection $(m^*, l^*) = (m_1^*, \ldots, m_N^*, l_1^*, \ldots, l_N^*)$ such that if p is determined by the rule above, then $\pi_i^*(p)$ is maximized given (m_{-i}^*, l_{-i}^*) by the choice (m_i^*, l_i^*), for each $i = 1, \ldots, N$. If each (m_i^*, l_i^*) actually maximizes $\pi_i^*(p)$ whether or not $(m_{-i}, l_{-i}) = (m_{-i}^*, l_{-i}^*)$, i.e., is a (weakly) dominant strategy, then we have a *Full equilibrium* (FE) of the game.

An interpretation of the pricing game in terms of a fictitious institution may clarify ideas. In a 'Dutch Sellers' (DS) auction,[13] the pointer on a 'price clock' moves steadily downward from a (sufficiently high) initial value and determines the prevailing price p_0 and individual sellers' posted prices p_i as follows. Each seller observes the clock and partially controls it with a button he can push twice. The first push (corresponding to l_i) indicates that the seller wishes to see p_0 fall no further; the pointer stops when *each* seller has pushed his button at least once (or at zero, if some seller never pushes his button) and its final reading defines p_0. A second push of the button (corresponding to m_i) indicates that seller i no longer wishes to match the prevailing price, and p_i is set at the clock's reading at that point. If a seller i never pushes his button a second time, then $p_i = p_0$.

A moment's reflection on the DS auction reveals that a producer cannot improve on the strategy of pushing his button for the first time when the

[10] See the opening paragraphs of Schmalensee (1976) for the argument that non-price competition is exempt from this criticism; i.e., that the Cournot-like ME concept is adequate. Also recall that I regard s_i as not directly observable by other producers. The key point, however, is that small changes in p_j (but not in s_j) can have large effects on π_i.

[11] Among these are Quick Response Equilibrium of Anderson (1984), the 'convolutions' of Marschak and Selten (1978), and the Consistent Conjectures Equilibrium of Bresnahan (1981). See Friedman (1984, pp. 27–29) for a brief justification of the present approach. It should be stressed that each of these approaches include my FE p_1^* among its equilibria.

[12] In formal terms, then, the producers actually play a three-stage extensive form game, with non-negative plays m_i, l_i and s_i in the stages pricing 1, pricing 2 and marketing. The equilibrium concept is subgame perfect NE.

[13] See Cassaday (1967, p. 193 ff) for a description of somewhat similar buyers' auctions.

clock pointer has declined to his preferred price p_i^*, and not pushing it a second time[14] until after it has declined to his entry price p_i^e. Thus the following Proposition should come as no surprise. A proof appears in the appendix.

Proposition 3 If (m^*, l^*) is a NE, then the prevailing price p_0 lies in the interval $[c_1, p_1^*]$, and posted prices p_i coincide with p_0 for each i such that $p_0 > p_i^e$. If (m^*, l^*) is actually an FE, then $p_0 = p_1^*$. The choice $(m_i, l_i) = (0, p_i^*)$ for $i = 1, \ldots, N$ is a FE.

Note that this Proposition establishes existence of FE (and hence NE) by explicit example, and establishes the uniqueness of FE prices. In conjunction with the previous Propositions, then, one can obtain explicit characterizations of FE sales costs, market shares and profits in terms of production cost and demand parameters.

There are several reasons one should regard the FE outcome as more significant than other NE outcomes. First, of course, is the presumption that if a simple (weakly) dominant strategy is available, it will be selected over a strategy that is only sometimes a 'best response'. Second, one observes that the FE outcome yields at least as high profts (and strictly higher profits for incumbents) as any other NE outcome, i.e., it is Pareto-superior. Finally, other outcomes seem relatively fragile. For example, one can sustain $p_0 \in [c_1, c_2)$ only if non-incumbents (who post $p_i = p_0$ but set $s_i = 0$) frighten the sole incumbent #1 with the prospect that they will take over the market if he raises his price p_1 above p_0. The threat is empty: such non-incumbents would obtain negative profits in that event, and one would expect that p_0 would soon adjust upward. Such fragility is present (albeit in attenuated form) as long as $p_0 \leqq p_i^e < p_1^*$ for any $i \neq 1$.

Therefore I shall regard the FE outcome as *the* prediction of the model. In particular, the equilibrium prevailing price p_0 is the lowest preferred price among incumbent producers, viz. the lowest cost producer's preferred price, p_1^*. However, one is really justified in calling this producer 'the price leader' only the extent that his preferred price depends much more on his own circumstances than on those of his rivals. A quick examination of the model's comparative statics, i.e., the sensitivity of $p_0 = p_1^*$ to the cost and demand parameters, is therefore in order.

Recall that p_1^* is typically a solution of the first-order condition $\pi_1^{*\prime}(p_0) = 0$. In this case, one can explicitly obtain $\mathrm{d}p_1^*/\mathrm{d}c_i$ by implicitly differentiating the

[14] Recall from the discussion preceding Proposition 2 that p_i^* and p_i^e generally depend on the set of producers that have not exited from the auction. Hence these strategies require that 'second pushes' be public knowledge. On the other hand, it turms out that 'second pushes' are not required in equilibrium. In the statement of Proposition 3, p_i^* is defined relative to the full set of producers.

bracketed expression in (7), but the results are rather messy. It is clear from inspection of (7), however, that small increases in the price leader's cost c_i increase the equilibrium price directly through the middle term[15] $E(p_0 - c_i)/p_0$, as well as through the v_1 and $-v_1$ appearing respectively in the numerator and denominator of the last ('market share') term. On the other hand, small changes in c_i, $i \neq 1$, exert only an indirect effect on price through the expressions $-v_T^{(2)}/v_T$ and $v_T/(n-1)$ in the market share term. Inasmuch as the sign of this indirect effect is ambiguous, and the last two expressions are relatively insensitive to c_i, my characterization of producer #1 as the price leader seems justified.[16]

Two qualifications are in order: (1) if (as may sometimes be the case) the price leader's preferred price is equal to the entry price for some 'marginal' producer j, then small changes in c_j rather than in c_1 will dominate changes in equilibrium price, according to the model. (2) Cost changes large enough to affect entry or exit decisions, or to yield a new price leader, obviously have a more complex effect on equilibrium price.

Finally, note that Q doesn't enter into the computation of p_j^*, so equilibrium price is insensitive to (elasticity-preserving) demand shifts. Of course, by formula (6), sales expenditures and profits will respond proportionately to demand shifts.

4. Discussion

I can summarize my argument as follows. For a homogeneous good not subject to price discrimination produced by heterogeneous firms (differing in production costs) that market their own output, one can expect efficient price searching by consumers to lead all producers to announce the same 'prevailing' price, with sales promotion being the most evident form of competitive activity. In non-collusive equilibrium, the prevailing price is governed by the 'price leader', a producer whose production costs lead him to desire the lowest prevailing price. In the simple case analyzed in this paper, one can unambiguously identify the price leader as the lowest-cost producer, and derive comparative static results.

What does the formal analysis have to say about the behavior of actual

[15] Indeed, one can show that this effect by itself would lead to a proportional mark-up rule, as for a simple monopolist. Combining this with the direct effects arising from the market share term, one can conclude that a small percentage increase in c_1 generally leads to a slightly *larger* percentage increase in equilibrium price.

[16] A simple numerical example may be in order. For a triopoly facing demand $Q(P_0) = 100p_0^{-1.5}$ with very similar marginal costs c_i of 1.00, 1.05 and 1.10, a numerical simulation indicates an FE price $P_1^* \doteq 2.31$ (for comparison, the other preferred prices $P_2^* \doteq 3.10$, $P_3^* \doteq 3.90$ are near or above the cartel price). A 0.01 cost increase for #1, the 'price leader', raises the FE price by about 0.15, while the same cost increase for either of the other producers lowers the FE price by about 0.05. When more producers are active and costs are more widely dispersed, one would expect a greater asymmetry.

markets, say the market for computer chips?[17] The comparative statics suggest that if the low-cost producer (NEC, say) finds a way to decrease unit cost slightly, NEC will lower its prices more than proportionately and obtain a larger market share at lower sales costs for everyone. On the other hand, if a higher cost producer (TI, say) obtains a similar cost reduction, TI optimally will not try to undercut the prevailing price, but will rather increase promotional activities and obtain a larger market share. (TI's rivals will in equilibrium also increase promotional activities, but to a lesser degree.) Although it does not appear explicitly in our (FE) equilibrium formulas, there is an asymmetry in the price mechanism when it comes to NEC's cost increases: rivals are under less immediate pressure to match its price increases (although it is still in their interest to do so) because the old FE prevailing price remains a NE after the cost increase. Hence prices may actually be more 'sticky' upwards (in the sense of less frequent but larger price changes) than downwards!

At this juncture it is worth asking how sensitive the model's rather sharp predictions are to modifications of the assumptions. In Friedman (1984, section 3), I relax the assumptions of constant production cost and proportional market shares, referred to as the 'standard case' assumptions in that paper. Given suitable generalizations of the concepts employed here (e.g., one must specify what happens to demand if there are 'shortages', which can arise given capacity constraints), the basic results obtained here remain valid, with the major exceptions that the lowest marginal cost producer is not necessarily the price leader (i.e., the producer with lowest preferred price), and the comparative statics become more complex – e.g., demand shifts *can* affect the FE price given capacity constraints. However, price responses certainly remain sticky and asymmetric.

More fundamentally, what happens if one allows some heterogeneity among the producers' goods, or buyer trade-off between price and convenience, or price discrimination by producers? Anderson (1984), employing an independently conceived but related model, finds that his results go through as long as the profit functions corresponding to my $\pi_i^*(p)$ obey a certain condition that would be satisfied if (for instance) goods differed according to fixed quality differentials. Of course reasonably complete answers to these questions require a model that derives demand for each producer from underlying preferences. Some preliminary work indicates that the main results survive in a spatial model with a fixed structure of transportation costs, and also as a limiting case in more general preference models. However, much work remains to be done to gain a theoretical understanding of more general producers' markets.

[17] Of course, dynamic considerations may also be important for an industry in which cost innovations are frequent and anticipated. The remarks in this paragraph are intended largely to suggest empirical applications and tests of the simple model.

To return to the opening theme, one could ask when a producer's market structure will arise, rather than some more nearly 'Walrasian' structure, such as one dominated by independent brokers. It should be noted that total sales costs are *not* closely related to intrinsic transactions costs in the present model, but respond rather sensitively and erratically to the distribution of production costs. In particular, there will be circumstances in which sales costs are mostly 'rent-seeking' by producers, and consumers (and perhaps even some producers) would be better off under an alternative market structure. The general question of which circumstances lead to the existence or relative efficiency of a given market mode is pivotal and remains wide open.

Appendix: Proofs

Lemma 1. Inequality (5) holds for all $k \leqq n(p_0)$. The function $n(p_0)$ is piecewise constant and non-decreasing, and $n(p_0) = N$ for $p_0 > p_N^e$. The entry prices satisfy $c_k \leqq p_k^e \leqq c_k + (k-1)(c_k - c_1)$ for all $k = 1, \ldots, N$; in particular $p_k^e = c_k$ for $k = 1, 2$.

Proof Note that $0 \leqq w_1 \leqq w_2 \leqq \cdots \leqq w_n$ if $Q(p_0) > 0$ and $c_1 \leqq c_2 \leqq \cdots c_n \leqq p_0$, so if (5) fails for some $k < N$, we have

$$\sum_{j=1}^{k+1} w_j = \sum_{j=1}^{k} w_j + w_{k+1} < (k-1)w_k + w_{k+1} \leqq [(k+1)-1]w_{k+1},$$

and (5) fails for $k+1$. Thus (5) holds for all $k \leqq n(p_0)$ and (by definition) fails for all $k > n(p_0)$.

The continuity of w_i in $p_0 > c_i$ follows from the continuity of demand Q, and yields continuity of $n(p_0)$ except at the entry prices p_k^e. Since $n(p_0)$ is integer-valued, it is piecewise constant. Since the RHS of (5) decreases more rapidly in p_0 than the LHS where the inequality holds, it follows that $n(p_0)$ is non-decreasing. Since $w_i/w_j \to 1$ as $p_0 \to \infty$, (5) holds for all p_0 sufficiently large; that is (by definition) for $p_0 \geqq p_N^e$. Note that (5) holds trivially for $k = 1, 2$, and $p_0 \geqq c_2$, so $p_1^e = c_1$ (by convention) and $p_2^e = c_2$. Finally, observe that $kw_1 \leqq \sum_{j=1}^{k} w_j$ so $kw_1 \geqq (k-1)w_k$ is sufficient for (5). But, from the definitions of w_1 and w_k, this last inequality is equivalent to $k(p_0 - c_k) \geqq (k-1)(p_0 - c_1)$, or $p_0 \geqq (k-1)(c_k - c_1) + c_k$. Thus $p_k^e \leqq (k-1)(c_k - c_1) + c_k$. An analogous argument yields $p_k^e \geqq (k-1)(c_k - c_{k-1}) + c_k \geqq c_k$. *Q.E.D.*

Proof of Proposition 1. From Lemma 1, one can readily verify that S_i^* as defined in (6) is positive, and straightforward computations show that it is equal to s_i^{**}/s_T^{**}, and that $s_T^{**} = (n-1)Q(p_0)/v_T$. Then it is easy to check that (2) holds if $p_i = p_0 > p_i^e$. Otherwise, one uses inequality (5) to conclude that

$\partial_i S_i(0, s_{-i}^{**}) = s_T^{**}/(s_T^{**})^2 = v_T/((n-1)Q(p_o)) < w_i$ if $p_i = p_0 \leqq p_i^e$, and of course $\partial_i S_i(0, s_i^{**}) = 0 < w_i$ if $p_i > p_0$; in either case, the Kuhn–Tucker condition (1) is satisfied for $s_i = 0$. Hence the specified values for $s^{**}(p)$ represent a simultaneous solution for (1) which must be unique by the argument preceding eq. (4). That $s^{**}(p)$ is indeed a marketing equilibrium will follow if the expression in (6) represents a profit maximum (rather than a minimum), since the Kuhn–Tucker solution is unique, and profits become negative for s_i sufficiently large (e.g., $s_i > p_0 Q(p_0)$ certainly implies $\pi_i < 0$) but are zero for $s_i = 0$.

Since π_i^* as specified in (6) is clearly positive (recall $p_0 > p_i^e \geqq c_i$ here), one must only verify the formula to complete the proof. In the following computation, $w_{T_p} := v_T/Q(p_0)$.

$$\pi_i^*(p_0) := (p_0 - c_i)Q(p_0)S_i(s^{**}) - s_i^{**}$$

$$= w_i^{-1} s_i^{**}/s_T^{**} - s_i^{**} = s_i^{**}\left(\frac{1}{w_i s_T^{**}} - 1\right)$$

$$= \frac{n-1}{w_{T_p}}\left(1 - \frac{n-1}{w_{T_p}} w_i\right)\left[\frac{w_{T_p}}{w_i(n-1)} - 1\right]$$

$$= \left(1 - \frac{n-1}{w_{T_p}} w_i\right)\frac{1}{w_i}\left(1 - \frac{n-1}{w_{T_p}} w_i\right) = \frac{1}{w_i}(S_i^*)^2$$

$$= (S_i^*)^2 (p - c_i) Q(p_0). \qquad Q.E.D.$$

Proof of Proposition 2. By perhaps deleting some producers from the original list, we may assume that $p_i = p_0$ for $i = 1, \ldots, N$, that $0 < c_1 \leqq c_2 \leqq \cdots \leqq c_N$, and that $p_N^e < p^c$, where $p^c = \sup\{p_0 : Q(p_0) > 0\}$ is the 'choke price', possibly ∞. Thus for $v_i := (p - c_i)^{-1}$ we have $0 < v_1 \leqq v_2 < \cdots \leqq v_k$ if $p_0 \geqq p_k^e \geqq c_k$. Now $n := \max\{k : p_0 \geqq p_k^e\}$ and $v_T := \sum_{i=1}^n v_j$ are both positive and piecewise continuous in p_0, with jump discontinuities at $p_0 = p_j^e$, $j \geqq 3$, by Lemma 1. However, the quotient $(n-1)/v_T$ is continuous, since (by inequality (5) of the text) both discontinuities are fractional increases of $(j-2)^{-1}$. Hence all the functions specified in Proposition 1 are continuous, and are differentiable except perhaps for $p_0 \in \{p_i^e, p_{i+1}^e, \ldots, p_N^e\} := D_i$.

For $p_0 \notin D_i$, we may compute, abbreviating S_i^* as S_i, $Q(p_0)$ as Q and $P_0 Q'/Q$ as $E < 0$,

$$\pi_i'(p_0) = QS_i^2 + (1/v_i)Q'S_i^2 + (2/v_i)QS_iS_i'$$

$$= QS_i^2\{1 + E/(p_0 v_i) + 2S_i'/(v_i S_i)\}.$$

Now,

$$S_i' = -(n-1)\left[\frac{v_i}{v_T}\right]' = -(n-1)\left[\frac{v_i'}{v_T} - \frac{v_i v_T'}{v_T^2}\right]$$

$$= \frac{-(n-1)}{v_T}\left[-v_i^2 - \frac{v_i(-v_T^{(2)})}{v_T}\right]$$

$$= \frac{n-1}{v_T} v_i \left[v_i - \frac{v_T^{(2)}}{v_T}\right],$$

where $v_T^{(k)} = \sum_{j=1}^{h} v_j^k$. Substituting this expression for S_i' and $S_i = 1-(n-1)v_i/v_T$ into { } and assuming $n>1$, one obtains eq. (7):

$$\pi_i'(p_0) = QS_i^2\left\{1 + \frac{E}{p_0 v_i} + 2\frac{v_i - v_T^{(2)}/v_T}{v_T/(n-1) - v_i}\right\}.$$

To establish existence of p_i^*, observe that as $p_0 \to \infty$ we have $p_0 Q(p_0) \to R(\infty) < \infty$ (by Assumption 5), $v_i/v_T \to 1/N$, and $S_i \to 1-(N-1)/N = 1/N$, so $\pi_i^* \to R(\infty)/N^2$. Since π_i is bounded and continuous on $[0,\infty]$, it must be maximized at some p_i^*. Formula (6) allows us to conclude that $\pi_i(p_0)>0$ iff $Q(p_0)>0$ and $p_0 > p_i^e$; hence $p^e < p_i^* \leqq p^c \leqq \infty$. In the case $R(\infty)=0$ (e.g., $p^c < \infty$) we obviously have $p_i^* < \infty$ for all i. However, if $R(\infty)>0$, then $E(\infty) = -1$ and the sign of π_i' is determined by the sign of S_i' for large p_0 (see the equation for π_i' preceding (7)). Since S_1' at least is always negative, we may conclude that $\pi_1' < 0$ for $p_0 < \infty$ sufficiently large, so $p_1^* < \infty$.

There remains only to show that the p_i^*'s have the proper ordering and are maximized on $[0, p_1^*]$ at p_1^*. Given the properties of π_i already derived, p_0^* will obviously be either ∞; a critical point (i.e., a solution to $\pi_i'(p_0)=0$); or a point in $D=\{p_2^e,\dots,p_N^e\}$ (in which case $\lim_{h\to 0}\pi_i'(p_i^* - h^2) \geqq 0 \geqq \lim_{h\to 0}\pi_i'(p_i^* + h^2)$. If p_i^* is the first (i.e., smallest p_0) relative maximum on $[p_i^e, p^c]$, then it is easy to see from (7) that $\pi_i' > 0$ on $[p_j^e, p_i^*)$ for $j \geqq i$, so $p_j^* \geqq p_i^*$, and the proof is complete in this case. However, if there are several relative maxima for π_i, and the global maximum does not occur at the first, the following Lemma suffices.

Lemma 2 Suppose p_1 and p_2 are relative maxima of π_j^*, and $\pi_j^*(p_1) \leqq \pi_j^*(p_2)$. If $k \geqq j$, then π_k^* is all maximized on $[p_1, p_2]$ at p_2.

Proof. To simplify notation, let π_i continue to denote π_i^*, S_i denote S_i^*, etc. Since π_i is continuous and piecewise differentiable, π_i is the integral of its derivative for each i, and it suffices to show $\int_p^{p_2} \pi_k'(p_0)\,\mathrm{d}p \geqq 0$ for all $p \in [p_1, p_2]$. Now for such p,

$$\int_p^{p_2} \pi_k'(p_0)\,\mathrm{d}p_0 = \int_p^{p_2} \pi_j'(p_0)\,\mathrm{d}p_0 + \int_p^{p_2} \pi_k'(p_0) - \pi_j'(p_0)\,\mathrm{d}p_0$$

$$\geqq \int_p^{p_2} \pi_k'(p_0) - \pi_j'(p_0)\,\mathrm{d}p_0 \tag{a}$$

$$\geqq \int_p^{p_2} QS_i^2 \{E/p_0(1/v_k - 1/v_j) + 2(v_k - t)/(\bar{v} - v_k)$$

$$- 2(v_j - t)/(\bar{v} - v_j)\}\,\mathrm{d}p_0 \geqq 0. \tag{b}$$

The justification for (a) is that π_j is by hypothesis maximized at p_2 so the integral of π_j' must be non-negative. For (b), I first employed $0 \leqq S_k \leqq S_j$ from Proposition 1; S_i refers to S_k if $v_j < t$ and to S_j in the opposite case. Then I used $E < 0$ and $v_k \geqq v_j$ to conclude that the first term in $\{\ \}$ is non-negative, and the inequalities

$$\frac{v_k - t}{\bar{v} - v_k} \geqq \frac{v_k - t}{\bar{v} - v_j} \geqq \frac{v_j - t}{\bar{v} - v_k}$$

to obtain a non-negative second term. *Q.E.D.*

Proof of Proposition 3. First note that $l = l(m) = l_1(m_1, \ldots, m_N), \ldots, l_N(m_1, \ldots, m_N)$ in general; that is, l_i is picked in full knowledge of which producers will match a price p_0 (and higher prices). Thus the set of ('potential incumbent') producers required to define p_i^e and p_i^* can be deduced from m; we will employ the notation $p_i^e(m)$ and $p_i^*(m)$ where appropriate in this proof to make this dependence explicit.

The first part of the Proposition is proved by contradiction. Assume that $(m^*, l^*(m)$ is a NE and $p_0 := \min_{j=1,\ldots,N} l_j^*(m^*) > p_1^*(m^*)$. Then from the definition of p^*, producer #1 could increase profits by reducing $l_1^*(m^*)$ to p_1^*, contradicting the hypothesis on p_0. The supposition that $p_1^*(m^*) > p_1^*(\underline{0}) := p_1^*$ also leads to a contradiction: in this case there must be some i who could earn positive profits at the prevailing price of p_1^* but has set $m_i^* > p_1^*$ and guaranteed $\pi_i = 0$, so m_i^* is *not* a best response. Finally, if $p_0 < c_1$, then the demand $Q(p_0)$ is positive and satisfied by a producer (or producers) whose unit cost exceeds prices. Such producers obtain $\pi_i < 0$, which is impossible in NE since setting $m_i = l_i$ sufficiently high can guarantee $\pi_i = 0$. Hence

$p_0 \in [c_1, p_1^*]$ in NE. That $p_i = p_0$ if $p_0 > p_i^e(m)$ in any best response is immediate from Proposition 1, and consequently $p_i^e(m) = p_i^e(\underline{0}) := p_i^e$.

The rest of the Proposition can be proved directly. Since the best i can do if $p_0 \in [0, p_i^e]$ is obtain $\pi_i = 0$, producer i will be indifferent among choices of m_i in $[0, p_i^e]$. However, a choice of $m_i > p_i^e$ will yield $\pi_i = 0$ rather than $\pi_i > 0$ whenever (m_{-1}, l_{-i}) yields $p_0 \in (p_i^e, m_i)$. Hence $m_i^* \in [0, p_i^e]$ is (weakly) dominant, and $l^*(m) = p_i^*(m)$ is also dominant by definition of preferred prices and Proposition 2. Therefore $(m, l) = (\underline{0}, p^*)$ is a FE, and for any FE(l^*, m^*), $p_0 := \min l_i^*(m^*) = \min p_i^*(m^*) = p_1^*(m^*) = p_1^*(0) = p_1^*$. *Q.E.D.*

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