ECON 204B Cheat Sheet for Midterm

Decision Theory

Expected Utility Hypothesis
$\max_{a\in A} E_\pi u = \max_{a\in A} \sum_{s\in S} \pi(s)u[x(a,s)]$
CRRA
$u(x|r) = \frac{x^{1-r}}{1-r}$
CARA
$u(x|a) = 1 - e^{-ax}$

Coefficient of Absolute Risk Aversion
$A(x) = -\frac{u\prime(x)}{u''(x)}$ $x \in (-\infty, \infty)$
Coefficient of Relative Risk Aversion
$R(x) = \frac{-u''(x)}{u'(x)}x = xA(x)$

Risk Measurement
$Var_L[x] = E_L(x - E_L x)^2$ Variance
$= \sum_{i=1}^N p_i(x_i - \bar{x})^2$ Variance
$\bar{x} = E_L x = \sum_{i=1}^N p_i x_i$ Mean
$\sigma_L = \sqrt{\text{Var}_L}$ Std Err.

Stochastic Dominance
$F(x) \geq G(x) \quad \forall x \Rightarrow$ G FOSDs F
$\mu_F = \mu_G, \int_{-\infty}^x F(t)dt \geq \int_{-\infty}^x G(t)dt \forall x \Rightarrow$ G SOSDs F

Definitions and Conditional Probabilities
$p(s) = \sum_{z\in Z} p(s,z)$ Prior prob. of s
$p(z) = \sum_{s\in S} p(s,z)$ Prob. of message z
$p(z|s) = \frac{p(s,z)}{p(s)}$ Likelihood
$p(s|z) = \frac{p(s,z)}{p(z)}$ Posterior Probability

Bayes Theorem
$p(s|z) = \frac{p(z|s)p(s)}{p(z)}$
$p(s|z) = \frac{p(z|s)p(s)}{\sum_{t\in S} p(z|t)p(t))}$
$\frac{p(s|z)}{p(t|z)} = \frac{p(z|s)p(s)}{p(z|t)p(t)}$
$ln(\frac{p(s|z)}{p(t|z)}) = ln(\frac{p(z|s)}{p(z|t)}) + ln\frac{p(s)}{p(t)}$

Value of Information
$VI = \sum_{z\in Z} p(z) \sum_{s\in S} p(s|z)[u_z^*(s) - u_0^*(s)]$ (Risk Neutral)
solves $0 = \sum_{z\in Z} \sum_{s\in S} p(z)p(s|z)u(a_z^* - x, s)$ (General Case)

Dynamic Programming
$V(y_0, 0) = \underset{a_t\in A_t}{\text{maximize}} \quad \sum_{t=0}^T F(a_t, y_t, t)$
s.t $\quad y_{t+1} = y_t + Q(a_t, y_t, t)$
$\quad G(a_t, y_t, t) \geq 0$

Game Theory Basics

Strategy s_i is *weakly dominant* if:
$f_i(s_i, s_{-i}) \geq f_i(s_i', s_{-i}) \quad \forall s_i' \in S_i, \quad s_{-i} \in S_{-i}$
A pure strategy $s_i \in B_i(s_i)$ is a *best response* to profile s_{-i} if:
$f_i(s_i, s_{-i}) \geq f_i(s_i', s_{-i}) \quad \forall s_i' \in S_i$

Nash Equilibrium
A profile $s_i^* = (s_1^*, ..., s_n^*)$ is a Nash Equilibrium if:
$s_i^* \in B(s_{-i}^*) \quad i = 1, ..., n)$

Result: Normal Form Game Solution
Start with a NFG or EFG;
while *there exists a dominant strategy* **do**
  Eliminate strictly dominated strategies and reduce the game;
  if *Only one profile remains* **then**
    it is the DS solution;
  end
end
if *Only one profile remains* **then**
  it is the IDDS solution;
end
Inspect for mutual BR → These are pure NE;
Check for mixed NE $\sigma_i \in B_i(\sigma_{-i})$;

Algorithm 1: Finding NFG Solutions

Payoff Function for Mixed NE
Player i's expected payoff when other players' pure strategy is s_{-i}:

$$f_i(\sigma_i, s_{-i}) = \sum_{k=1}^N p_k f_i(t_k, s_{-i})$$

Player i's expected payoff when playing own pure strategy t_i:

$$f_i(t_k, \sigma_{-i}) = \sum_{j=1}^m q_j f_i(t_k, s_{-i}^j)$$

This is used to find mixed NE. For example, in the 2×2 case:

$$f_1(s_1, \sigma_{-1}) = f_1(s_2, \sigma_{-1})$$

$$f_2(s_1, \sigma_{-2}) = f_2(s_2, \sigma_{-2})$$

Extensive Form Games

Procedure for finding solution of EFGs with perfect information

1. Convert penultimate nodes ν into terminal nodes
   (a) If ν is owned by player i, then he chooses the maximum payoff
   (b) If ν is owned by Nature, take expectation over the payoff vector
2. Iterate over step 1 until reaching the initial node
3. Reconstruct each player's strategy for their choices in steps 1-2
4. The resulting profile is subgame perfect Nash equilibrium (SPNE)
5. In the case if *imperfect information*, find the smallest subgame that contains terminal nodes. Find all NE of that subgame. Replace the initial node of that subgame by a NE payoff vector. Iterate to a solution and get one SPNE. Then loop on step 1 until all N equilibria in the minimal game have been used.

Bayesian Nash Equilibrium, PBE, and Seq. EQ

1. Beliefs μ_i at each info set for player i are consistent with *common prior* and likelihood from s_{-i}^* and own realized type $\bar{\theta}_i$ via Bayes
2. At each info set, player i maximizes $E(u_i|\mu_i)$:

$$E_{\theta_i}[u_i(s_i^*(\bar{\theta}_i), s_{-i}^*(\theta_{-i}), \theta_i)|\bar{\theta}_i \geq E_{\theta_i}[u_i(s_i'(\bar{\theta}_i), s_{-i}^*(\theta_{-i}), \theta_i)|\bar{\theta}_i]$$

3. Previous items hold in every subgame
4. Robust to sufficiently small trembles

- 1 and 2 constitute a *Bayesian Nash Equilibrium*
- 1 - 3 constitute a *Perfect Bayesian NE*
- 1 - 4 constitute a *Sequential Equilibrium*