1 Utility

EUH For each person, $\exists$ Bernoulli function $u(\cdot)$, s.t. $L \succ M \iff E_L u > E_M u$ and always $\max_{a\in A} E_\pi u$

CRRA Utility $u(x|r) = \frac{x^{1-r}}{1-r}$, $r \in (-\infty, \infty)$

CARA Utility $u(x|a) = 1 - e^{-ax}$, $a > 0$

coefficient of relative risk aversion

$R(x) = xA(x) = -\frac{xu''(x)}{u'(x)}$

coefficient of absolute risk aversion

$A(x) = -\frac{u''(x)}{u'(x)}$ *(A(x)↑ risk averse ↑)*

Certainty Equivalent $u(CE) = \int u(x)\, dF(x)$

Risk Premium $u(\int x\, dF(x) - RP) = \int u(x)\, dF(x)$

Mean-Variance Approximation

$u(\overline{x} + h) = u(\overline{x}) + (x - \overline{x})u'(\overline{x}) + \frac{1}{2}(x - \overline{x})^2 u''(\overline{x}) + R^3$

$Eu(x) = u(\overline{x}) - \frac{1}{2}A(\overline{x})\sigma_L^2 + ER^3$

First Order Stochastic Dominance

G FOSDs F if $F(x) \geq G(x)\ \forall x$

Second Order Stochastic Dominance

G SOSDs F if $\mu_F = \mu_G$ and $\int_{-\infty}^{x} F(t)dt \geq \int_{-\infty}^{x} G(t)dt\ \forall x$

2 Bayes' Theorem

Basic Definitions

$p(s) \equiv$prior prob. of state s; $p(z) \equiv$ message prob.

$p(z|s) \equiv$ likelihood ; $p(s|z) \equiv$ posterior prob.

Bayes theorem

(i) $p(s|z) = \frac{p(z|s)p(s)}{p(z)} = \frac{p(z|s)p(s)}{\sum_{t\in S} p(z|t)p(t)}$

(ii) $\frac{p(s|z)}{p(t|z)} = \frac{p(z|s)p(s)}{p(z|t)p(t)}$

(iii) $\ln\frac{p(s|z)}{p(t|z)} = \ln\frac{p(z|s)}{p(z|t)} + \ln\frac{p(s)}{p(t)}$

Conditional independence

$p(z_1, z_2|s) = p(z_1|s)p(z_2|s), \forall z_1 \in Z_1, z_2 \in Z_2, s \in S$

Decision Tree

BI: nature node →taking expected value;

Player node→taking maximum

Value of information

Risk Neutual: $V_I = \sum_{z\in Z} p(z) \sum_{s\in S} p(s|z)[u_z^*(s) - u_0^*(s)]$

3 Dynamic Programming Problem

$$V(y_t, t) = \sup_{a_t\in A_t} \{F(a_t, y_t, t) + V(y_{t+1}, t+1)\}, t = 0, ..., T-1$$

Bellman equation for stochastic problem

$$V(y) = \sup_{a_t\in A_t, t>t_0} \{F(a_t, y_t) + d\sum_{y\in S} V(y)p(y|y_{t_0}, \alpha_t)\}$$

4 Normal Form Games

Cookbook for NFG solutions

(i) Get NFG from story or EFG (should be a **complete** contingency plan)

(ii) Eliminate strictly dominated strategies. If only one profile remains, it is DS solution

(iii) Iterate step(i) until no more dominated strategies. If only one profile remains, it is IDDS

(iv) Inspect for mutual BR ⟶ These are pure NE

(v) Check for mixed NE, $\sigma_i \in B_i(\sigma_{-i})$, each $|subset| \geq 2$ of pure strategies for each player, write down the set of simultaneous equation

Payoff function of mixed strategies (2x2)

$$f_1(\sigma_1, \sigma_{-1}) = \sum_{i=1}^{2} p_i \sum_{j=1}^{2} q_j f_1(s_i, t_j)$$

$$where\, \sigma_1 = p_1 s_1 + (1 - p_1)s_2, \sigma_{-1} = q_1 t_1 + (1 - q_1)t_2$$

Formula for finding mixed strategies (2x2)

$$f_1(s_1, \sigma_{-1}) = f_1(s_2, \sigma_{-1})$$

$$f_2(t_1, \sigma_{-2}) = f_2(t_2, \sigma_{-2})$$

Correlated equilibrium is a distribution p^* over action profiles a s.t. each i best responds to the conditional dist.: $\forall j \in supp(p_i^*)$,

$$\sum_{a_{-i}\in A_{-i}} u_i(j, a_{-i})p(a_{-i}|j) \geq \sum_{a_{-i}\in A_{-i}} u_i(j', a_{-i})p(a_{-i}|j)$$

Relations: Corr Eq. ⊃ NE ⊃ IDDS ⊃ DS

5 Extensive Form Games(Trees)

Cookbook for perfect information

(i) Use BI starting from penultimate nodes; iterate until you reach the initial node

(ii) Reconstruct each player's strategy from (i)

(iii) The resulting profile is a subgame perfect nash equilibrium

Cookbook for imperfect information

(i) Find a smallest subgame that contains terminal nodes. Find all NE of that subgame.

(ii) Replace the initial node of that subgame by a NE payoff vector. Iterate to a solution ⟶ get one SPNE.

(iii) Re-do step (ii) using a different NE, until all NEs in all minimal subgames have been used to get the other SPNEs.

Harsanyi Cookbook

(i) Encapsulate the incomplete information as a set of types for one or more players.

(ii) Specify type contingent games and tie them together by an initial Nature move and appropriate info. sets.

(iii) Assign probs(common prior) of Nature's initial move.

(iv) Solve the game for NE and subgame perfect NE, keeping track of all the relevant probabilities via Bayes' rule.

6 BNE, PBE, Seq Eq

(i) Beliefs μ_i at each info set for player i are Bayes-consistent with common prior, likelihoods from s_{-i}^* and own realized type $\bar{\theta}_i$.

(ii) At each info set, player i max's $E(u_i|\mu_i)$: $\forall\ s_i' \epsilon S_i$

$E_{\theta_{-i}}[u_i(s_i^*(\bar{\theta}_i), s_{-i}^*(\theta_{-i}), \bar{\theta}_i)|\bar{\theta}_i] \geq E_{\theta_{-i}}[u_i(s_i', s_{-i}^*(\theta_{-i}), \bar{\theta}_i)|\bar{\theta}_i]$

(iii) previous items hold in every subgame

(iv) robust to sufficiently small trembles

(i) and (ii) constitute a Bayesian NE

(i) thru (iii) constitute a Perfect Bayesian NE

(i) thru (iv) constitute a sequential equilibrium