7. Monopoly

Back to Varian! Ch 14 covers almost everything in these notes, and more!

I. Simple Monopoly vs. Perfect Competition

A. The core behavioral rule is just the same...

The firm maximizes profit.

$\max py - c(y)$

B. The resulting decision rule is the same...

The firm still sets marginal revenue equal to marginal cost.

C. But the things the firm has control over have now changed...

We no longer assume that the firm is a price taker.

1. Mathematically, this means that there is some functional relationship between the quantity the monopolist produces and the price the monopolist can charge.
2. The firm's problem is now subtly different:

   $\max p(y)y - c(y)$

3. In particular, marginal revenue is not pegged to some exogenously given price.

D. Thus monopolists and competitive firms differ on the revenue side.

1. When a competitive firm decides to change output, it only has to think about one effect.
   a. Revenue increases by $p\ dy$

2. Monopolists have to trade off two effects:

   a. Revenue *increases* by $p\ dy$.

   b. In order to sell more units, the monopolist has to lower price.

      - Revenue *decreases* by $ydp = y\frac{dp}{dy}dy$
      - Unless the monopolist can price discriminate, this revenue change affects all of the monopolist's units – not just the marginal ones.
      - These non-marginal units are called inframarginal.

3. Summing these effects we get an expression for marginal revenue:

$$\begin{aligned} dR &= d(py) = pdy + ydp = [p + \frac{dp}{dy}y]dy \\ \implies MR = \frac{dR}{dy} &= p(y) + \frac{dp}{dy}y \qquad (1) \\ &= p[1 + \frac{dp}{dy}\frac{y}{p}] \\ &= p[1 + \frac{1}{\epsilon_p}] \qquad (2) \end{aligned}$$

where $\epsilon_p = \frac{dy}{dp}\frac{p}{y}$ is own-price elasticity of demand.

II. Profit Maximization

A. The first order condition for the firm's problem sets marginal revenue equal marginal cost.

$$p(y)[1 + \frac{1}{\epsilon(y)}] = c'(y) \qquad (3)$$

Ex: Const Elastic Demand : use (3)

Ex: Linear Demand: use (1)

B. $|\epsilon(y)| > 1$ at the Monopolist's production.

1. Otherwise we'd have negative MR, which implies that reducing production increases profit.

III. Deadweight Loss

A. When firms are competitive, marginal costs of production equal (inverse) demand.

1. Marginal revenue is wherever the supply curve happens to intersect the demand curve.
2. This is a consequence of the price taking assumption.

B. If demand slopes downward, the monopolist's marginal revenue curve will lie below the demand curve.

$p(y) > p(y)[1 + \frac{1}{\epsilon(y)}]$

Since MC is upward sloping, we conclude that a monopolist will cut back output relative to a price-taker: $q^M < q^C$.

C. Since q^C is efficient (max TS), we conclude that monopoly pricing *must result in deadweight loss.*

Caveat: price discrimination can actually reverse this effect.

Ex: Quasilinear case.

IV. Passing Along Taxes (and Other Costs)

A. What happens to prices if the costs of production change?

B. To make things simple, we assume linear costs.

1. This is a very common assumption in industrial organization.
2. In this case it makes it easy to substitute cost increases for tax increases.

C. In this case it can be shown that

$$\frac{\partial p}{\partial c} = \frac{1}{2 + yp''(y)/p'(y)}. \tag{4}$$

D. So...

1. With linear demand, half of costs are passed on in the price.
2. With const elasticity demand, price goes up *more than proportionately* when costs increase! (see Varian, p.237.)

V. Where Do Monopolies Come From?

A. Natural Monopolies

1. A natural monopoly is an industry with a cost structure that prohibits marginal cost pricing.
   a. Competitive production and pricing would lead to negative profits.
   b. Regulation $p = MC$ would drive firm out of business.
2. Occurs when fixed costs are extremely large relative to marginal costs.
3. Typical regulatory solution is to force monopolists to price at average cost.
   a. Allow monopolists to just break even (i.e., normal return on capital).
   b. Still inefficient.
   c. Gives regulated firm an incentive to inflate costs.

Ex: Natural Monopoly

4. Efficient Scale: What level of output minimizes AC?
5. If there is enough demand in the market to accommodate multiple firms producing at this cost, then an industry can be competitive.
    a. Otherwise only a single firm can survive.
6. Notice that MES is determined by technology
    a. Regulated pricing in an industry could easily reduce incentives to invent new technologies that disrupt the MES.

B. Barriers to Entry

1. Explicit barriers
2. Raising rivals cost
3. Price controls
4. Product standards

C. Problem set asks: Which of these firms are natural or unnatural monopolists? What are reasonable public policies towards them? Google, Facebook, AirBnB, Amazon, Uber.

VI. Price Discrimination Overview

A. Price discrimination is the sale of identical units of a good at different prices.

B. By price discriminating, a firm can capture some of what would be consumer surplus. PS ↑.

1. In so doing, a monopolist may also increase output, leading to a more efficient outcome. So it is possible that we also have CS ↑.

C. Constraints on price discrimination

1. The firm must have market power; otherwise it will just be a price-taker.
2. Arbitrage must somehow be limited; otherwise low price units could be resold and undercut the higher priced units.
3. The firm must somehow be able to detect WTP differences across consumers, and/or across units purchased by a single customer.
4. There may also be legal or moral constraints.

D. The three classical types of price discrimination are methods of coping with the constraints and sorting consumers according to their WTP.

VII. A Basic Model (from Varian, mastery not required)

A. A simple quasilinear model helps explain several varieties of price discrimination.

B. Consumers, $i = 1, 2$ have utility $u_i(x) + y$, normalized so that $u_i(0) = 0$. .

1. Think of y as money left over for everything other than x.
2. Consumers are willing to pay up to $u_i(x)$ for x units of the good.
3. Hence i's marginal WTP is $u_i'(x)$.

C. The inverse demand curve for the individual consumer is therefore found by solving the consumer's problem

1. $\max u_i(x) + y$

   s.t. $px + y = m$

2. FOC is $p = u_i'(x)$, the inverse demand curve.
3. In other words, i's marginal WTP is $u_i'(x)$.

D. From now on we'll assume that consumer 2 has higher WTP than consumer 1.

1. $u_2'(x) > u_1'(x)$, so by integration,
2. $u_2(x) > u_1(x)$.
3. This implies the **single crossing property**, that the indifference curves of two consumers only cross once.

E. To avoid messy expressions, assume that the monopolist has constant marginal cost: $c(x) = cx$.

VIII. Perfect Price Discrimination (aka First Degree)

A. The monopolist is able to charge a different price for each unit sold.

B. To see the implications, suppose the monopolist offers each buyer i a package of exactly x_i units for a lump payment of r_i. She solves

$$max_{r \geq 0} r - cx \text{ s.t. } u_i(x) \geq r. \tag{5}$$

C. Constraint holds with equality at optimum, so $r^* = u(x^*)$.

D. The FOC is $u'(x^*) = c$ .... which is Pareto efficient!

... this x^* is the same level of output as a competitive firm, where $u'(x) = p(x) = c$.

... of course, the monopolist gets the entire maximized TS.

E. This lump sum solution is equivalent to charging a different price (for marginal willingness to pay) for each unit of the good.

F. Constraints: all of them are problematic here.

G. Colleges attempt to approximate this for families who apply for financial aid.

First degree price discrimination is not necessarily an evil plot by the producer. Given high fixed costs, it may be the only way for the producer to stay in business.

IX. Second Degree Price Discrimination

A. The monopolist charges prices that are not simple per-unit prices.

1. Sometimes called nonlinear pricing.
2. Includes quantity discounts, and block pricing as in local water bills.

   Also gold/silver/bronze data plans or health plans.

B. Simplest version: a monopolist offers two different price/quantity *bundles* (r_i, x_i)

1. Bundle i is designed for consumers of type i.
2. The monopolist doesn't know whether a given consumer is type 1 or type 2.
3. The pricing scheme encourages consumers *sort themselves*.

C. The following analysis is optional for MS students (but PhD students are supposed to master more general versions.)

D. In order to get type i consumers to choose the targeted bundle i, the monopolist has to satisfy two types of constraints:

1. Individual Rationality (aka participation) constraints:

   $u_1(x_1) - r_1 \geq 0$ ***

   $u_2(x_2) - r_2 \geq 0$

2. Self selection (aka. incentive) constraints:

   $u_1(x_1) - r_1 \geq u_1(x_2) - r_2$

   $u_2(x_2) - r_2 \geq u_2(x_1) - r_1$ ***

3. A profit maximizing producer wants to set r_1 and r_2 as high as she can while satisfying the constraints.

4. This fact combined with the single crossing property guarantees that some of the constraints above bind, i.e., hold with $=$, not with $>$. It turns out (see Varian) that the binding constraints are the two marked ***. Thus we have

   $r_1 = u_1(x_1)$

   $r_2 = u_2(x_2) - u_2(x_1) + r_1 = u_2(x_2) - u_2(x_1) + u_1(x_1) < u_2(x_2).$

5. That is, charge the low value consumer his max WTP for the low target bundle, and charge the high value consumer the highest price that doesn't cause him to switch away from the high target bundle.

E. The monopolist's problem

1. The monopolist gets the sum of the profits from the two consumer types.

   $\pi = r_1 - cx_1 + r_2 - cx_2$

2. All of our hard work above gives us constraints to substitute into this equation:

   $\pi = u_1(x_1) - cx_1 + u_2(x_2) - u_2(x_1) + u_1(x_1) - cx_2$

3. We can maximize this with respect to outputs x_1 and x_2, to obtain the FOCs below.

F. Welfare

1. The FOC $0 = \frac{\partial \pi}{\partial x_1}$ yields

$$u_1'(x_1) = p(x_1) = c + u_2'(x_1) - u_1'(x_1)$$

2. This tells us that the per unit price charged to the low value consumer is above marginal cost, implying implying a DWL.
3. The other first order condition, $0 = \frac{\partial \pi}{\partial x_2}$, yields

$$u_2'(x_2) = p(x_1) = c.$$

4. This tells us that the per unit price charged to the high value consumer is equal to marginal cost. No efficiency loss here.

G. Conclusion: To max profit, target a bundle to high value consumer (#2) s.t. price (on last unit) = MC. Find a bundle for low value consumer (#1) that cuts back from the efficient quantity, is (barely) not preferred by #2, but exhausts #1's WTP.

H. Remark. I am unaware of any firm that actually does such calculations to obtain their price/quantity menu. It's hard to estimate the preferences u_i, and estimation errors could throw the calculation way off. Yet this approach gives insight to the menu that firms may settle upon after trial and error.

X. Third Degree Price Discrimination

A. This case is the most realistic, and the calculations parallel the way some firms actually think about it.

B. The monopolist is able to charge different prices to identifiably different groups but not able to charge different prices within any group.

1. Think student discounts, or senior citizen discounts.

2. Or last minute shoppers, or domestic vs foreign market.

C. Assume for now that arbitrage is not possible.

1. The monopolist's problem is

   $\max\ p_1(x_1)x_1 - cx_1 + p_2(x_2)x_2 - cx_2$

2. The FOCs from this problem can be written as:

   $p_1(x_1)[1 - \frac{1}{|\epsilon_1|}] = c$

   $p_2(x_2)[1 - \frac{1}{|\epsilon_2|}] = c$

3. So we can write $p_i = M_i c$ where markup factor is $M_i = \frac{1}{1-\frac{1}{|\epsilon_i|}} = \frac{|\epsilon_i|}{|\epsilon_i|-1}$.

4. In particular, $p_1(x_1) > p_2(x_2)$ only when $|\epsilon_1| < |\epsilon_2|$

5. What if $|\epsilon_1| < |\epsilon_2|$ but arbitrage is possible, at unit cost k?

   a. If the formulas above give $p_1 \leq p_2 + k$, then arbitrage is unprofitable and has no impact.

   b. But if they give $p_1 \geq p_2 + k$, then profitable arbitrage will undermine that form of price discrimination.

   c. In that case, the profit-maximizing choice can be found by writing $p_2 = p$ and $p_1 = p + k$, then putting this into a profit function for the firm, and finding the profit-maximizing p.

D. Welfare

1. Does the ability to price discriminate in the third degree help or hurt social welfare (TS)?

2. This depends on the effects on output. Varian shows that it can go either way (or be neutral).
    a. First, the only way welfare can be *improved* is if output increases due to the discrimination.
    b. Second, if prices and output changes (relative to ordinary monopoly) satisfy $(p_1 - c)\Delta x_1 + (p_2 - c)\Delta x_2 > 0$, then welfare has to improve!
    c. Third, if a whole new market is served due to the discrimination, welfare has to improve.

XI. Other forms of price discrimination

- 2-part tariffs
    A. membership fee plus (low) per unit price.
    B. if all customers are identical, can obtain same result as perfect price discrimination by charging $p = c$ and fee=CS.
    C. behavioral economics angle: gym membership.
- bundling
    A. season tickets, package tours, ...
    B. can increase firm's profits to the extent that consumers differ in their WTP for bundle components.
    C. in that sense, covers the opposite case from 2-part tariffs.
- loyalty programs
    A. usually take the form of quantity discounts, with a time lag

B. frequent flyer miles are partially convertible to currency

C. generally try to blunt direct price competition

- peak-load pricing

A. can be thought of as market segmentation, but

B. there may also be differences on the cost side.

C. an extreme recent version: Texas utility makes electricity free after 9pm (NYT NOV. 8, 2015).