Answer Key for Final Exam

Econ 204b, UCSC Winter 2018

1. The characteristic function in a 3 player game assigns worth of 1 to the coalitions K=1,2, 1,3 and 1,2,3, and assigns worth 0 to all other coalitions.
a. Is this game convex? Explain as briefly as possible. (4pts)
No, not convex because the marginal contribution of some players is smaller in larger coalitions. E.g., $MC_2(123) = 0 < 1 = MC_2(12)$.

b. What is the Core of this game? (6pts)
Any core allocation (x_1, x_2, x_3) satisfies $x_1 + x_2 + x_3 = v(123) = 1$ and $x_1 + x_2 \geq v(12) = 1$, so $x_3 = 0$. Likewise, $x_1 + x_3 \geq v(13) = 1$, so $x_2 = 0$. Since $v(K) = 0$ for the other coalitions, there are no other constraints (blocked imputations). We conclude the core consists only of the single imputation $(x_1, x_2, x_3) = (1, 0, 0)$.
Comment. This is like a weighted majority voting game where player 1 is uniquely pivotal.

c. What is the Shapley Value of this game? (6pts)

CoalitionOrder	MC_1	MC_2	MC_3
123	0	1	0
132	0	0	1
213	1	0	0
231	1	0	0
312	1	0	0
321	1	0	0
Sum	4	1	1
Shapley value	2/3	1/6	1/6

Note that the Shapley value is not in the Core, but this is possible since the game is not convex.

2. Your company is seeking to begin operations in a new region where you have no local contacts. A large number of local residents have applied to be sales representatives for your company. You know that the best of them would be very productive, but others would not be at all productive, and that most are in between. Unfortunately you have no direct way to detect productivity before hiring. a. Use Econ 204b jargon to describe this situation, and to explain obstacles to obtaining mutual gains from hiring. (6pts)
Many answers are acceptable. There is asymmetric information in that the applicants know more about their abilities than you do. That is an obstacle since there can be adverse selection and market failure.

b. Name at least three standard ways that might (partially) overcome the obstacles. Which of them seem appropriate in this case? (6pts)
Again, many answers are possible. Three standard ways to overcome information asymmetries are (i) to destroy the private information or the opportunity to use it, (ii) for the informed party to find a credible signal (separating PBE) and (iii) for the uninformed party to screen by creating a menu in which the informed party self-selects. The latter option seems more appropriate here, e.g., offering a choice between a modest fixed wage (targeted at less productive applicants) and a plan that pays a large bonus for high productivity but a tiny base wage. Alternatively, hire temporarily and later offer a permanent job to the most productive workers; this can be construed as signalling.

3. Two companies produce standard tractors, Centipede (C) and Dearjohn (D), at respective marginal cost

$c_C = 2$, and $c_D = 3$. When their outputs are q_C and q_D, they experience inverse demand $p_C = 14 - 2q_C - q_D$, and $p_D = 15 - 3q_D - q_C$ respectively.

a. what can you infer about the substitutability (or complementability) of the two kinds of standard tractors (e.g., are they perfect substitutes)? (4pts)

They are substitutes because when quantity of one product increases, the price of the other product decreases, implying a decrease in demand. They are imperfect substitutes because own price impact is stronger than cross price impact.

b. Use Nash equilibrium to predict prices and quantities when both companies independently choose output quantities. (8pts)

C's problem is to maximize profit

$$\max_{q_C} \ (14 - 2q_C - q_D)q_C - 2q_C.$$

The first-order condition is

$$14 - 4q_C - q_D = 2. \tag{1}$$

In a similar way, the first-order condition for D is

$$15 - 6q_D - q_C = 3. \tag{2}$$

Solving equations (1-2) simultaneously gives the Cournot Nash equilibrium $(q_C^*, q_D^*) = (\frac{60}{23}, \frac{36}{23})$. Use the given inverse demand functions to obtain the corresponding prices $(p_C^*, p_D^*) = (\frac{166}{23}, \frac{177}{23})$.

c. Suppose that C can publicly and credibly commit to an output quantity q_C before D chooses q_D, and that D can make no such commitment. What output level q_C would maximize C's profit? (6pts)

This is a Stackeberg-Cournot problem. C anticipates that D will react according to equation (2), i.e., $12 = 6q_D + q_C \implies q_D = BR_D(q_C) = \frac{12 - q_C}{6} = 2 - \frac{q_C}{6}$. So C's problem is

$$\max_{q_C} \ (14 - 2q_C - (2 - \frac{q_C}{6}))q_C - 2q_C = (10 - \frac{11}{6}q_C)q_C$$

with first order condition $0 = 10 - \frac{22}{6}q_C \implies q_C = \frac{30}{11}$.

If you wish you can obtain q_D from the best response function, then prices from the given inverse demand functions, and then profits from the profit expressions used to obtain the original FOCs.

d. Now suppose that, contrary to previous parts of this problem, the companies simultaneously choose price and then produce to order. How do you find the Nash equilibrium prices and quantities? You can get full credit for this part of the problem just by describing clearly the steps you would take. (6pts)

e. If time permits, go ahead and compute the NE in part d. (extra credit, 4pts max)

This is a Bertrand equilibrium problem with imperfect substitutes. The first step is to obtain direct demand functions by solving the inverse demand functions simultaneously for the q's:

$$q_C = \frac{27}{5} - \frac{3}{5}p_C + \frac{1}{5}p_D \tag{3}$$

$$q_D = \frac{16}{5} - \frac{2}{5}p_D + \frac{1}{5}p_C \tag{4}$$

Next, maximize the profit functions

$$\max_{P_C} \pi_C(p_C, p_D) = (p_C - C_C)q_C = (p_C - 2)(\frac{27}{5} - \frac{3}{5}p_C + \frac{1}{5}p_D) \tag{5}$$

$$\max_{P_D} \pi_C(p_D, p_C) = (p_D - C_D)q_D = (p_D - 3)(\frac{16}{5} - \frac{2}{5}p_D + \frac{1}{5}p_C). \tag{6}$$

Obtain the FOCs

$$0 = \frac{33}{5} - \frac{6}{5}p_C + \frac{1}{5}p_D \implies p_C = \frac{11}{2} + \frac{1}{6}p_D \tag{7}$$

$$0 = \frac{22}{5} - \frac{4}{5}p_D + \frac{1}{5}p_C \implies p_D = \frac{11}{2} + \frac{1}{4}p_C. \tag{8}$$

(Note that these reaction functions are upward sloping (strategic complements) unlike the downward sloping Cournot BR functions (strategic substitutes).)

The Bertrand Nash equilibrium is the simultaneous solution to (7-8), namely $p_C = \frac{154}{23}$ and $p_D = \frac{165}{23}$. Compare prices, quantities and profits between Cournot and Nash. The difference is not as extreme in this example of imperfect substitutes as it is in the standard case of perfect substitutes.

4. An industry consists of a large population of firms, each of which must choose one of two alternative technologies. The first technology has decreasing returns to scale when rare and increasing returns when common; its profitability can be expressed as $u_1 = 2s_1^2 - 2s_1 + 1$, where s_1 is the fraction of industry output produced using that technology. Technology 2 has moderately decreasing returns at all scales; its profitability is $u_2 = 0.5(1.5 - s_2) = 0.5(s_1 + 0.5)$ when fraction $s_2 = 1 - s_1$ of the output is produced using it.

a. Write down the payoff difference $D = u_1 - u_2$ as a function of s_1, and graph this function. (4pts)

$D = 2s_1^2 - 2.5s_1 + 0.75 = 2(s_1 - 0.5)(s_1 - 0.75)$, with roots $s_1^* = 0.5, 0.75$, as shown in Figure 1.

b. Does this game have a pure strategy NE? I.e., is $s_1 = 0$ or 1 a NE? Please verify your answer. (6pts)

Figure 1 shows that $s_1 = 1$ is an ESS (or EE) and therefore a NE, and this can be seen directly since $D > 0$ at $s_1 = 1$, implying that technology 1 is the BR when universally adopted. Likewise, $s_1 = 0$ is a source, and so a NE; the direct argument is that $D > 0$ at $s_1 = 0$, implying that tech 1 is the BR when it has a tiny share, so technology 2 is not its own best response.

c. Suppose that sign preserving dynamics describe the evolution of technology adoption in the industry. Find the evolutionary equilibria and their basins of attraction, using the graph from part a. (6pts)

As shown in Figure 1, the steady states $s_1^* = 0.5$ and $s_1 = 1.0$ are stable because $\frac{d\Delta w}{ds_A}(0.5) = 4(0.5) - 2.5 = -0.5 < 0$ and $\Delta w(1) = 2(1)^2 - 2.5(1) + 0.75 = 0.25 > 0$. By contrast, the steady states $s_1^* = 0.75$ and $s_1 = 0$ are unstable, since $\frac{d\Delta w(0.75)}{ds_A} = 4(0.75) - 2.5 = 0.5 > 0$ and $\Delta w(0) = 2(0)^2 - 2.5(0) + 0.75 = 0.75 > 0$. Thus unstable steady state $s_1^* = 0.75$ separates the basins of attraction for stable steady states $s_1^* = 0.5$ and 1.0.

d. Suppose that the second technology is more recent. Predict the long-run shares of the two technologies. (4pts)

The supposition is that we begin with $s_1 = 1$, i.e., in its own basin of attraction. Then we stay in that basin, and the new (second) technology remains at zero share. By contrast, if technology 1 were newer, and so we began at $s_1 = 0$, then it would never be adopted without a large outside intervention that forced us into the other basin of attraction.

5. A nomadic Clan has to decide whether to Expel one of its members named Blaine. Nature made Blaine Strong with probability p and Weak with probability 1-p. Blaine knows his type but the Clan leadership does not. Blaine has the opportunity to make a demonstration to his peers by Ascending a sheer rock face, or he can chose to Not Ascend. The probability of successfully climbing the face is q if Blaine is strong and r if Blaine is weak. If the climb is unsuccessful, a Blaine of either type ends up Dead. Following either a successful

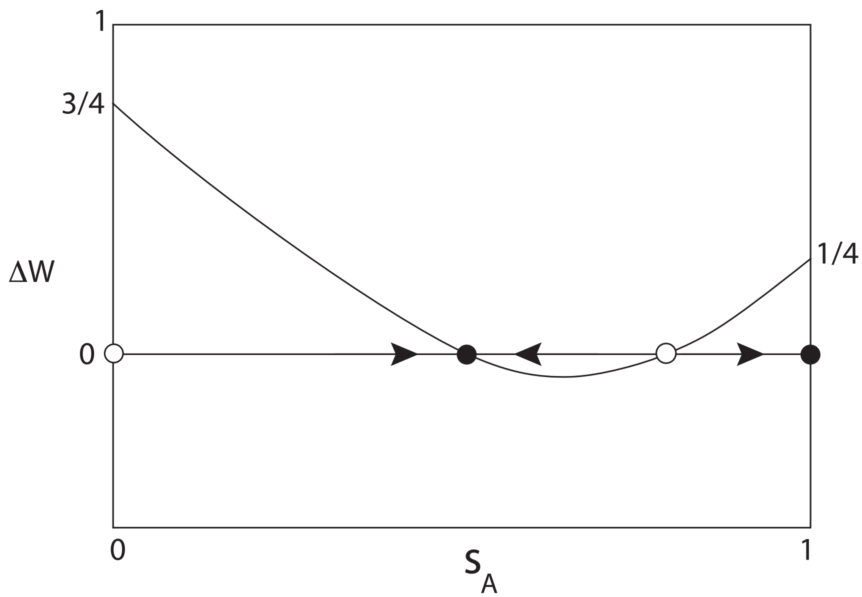


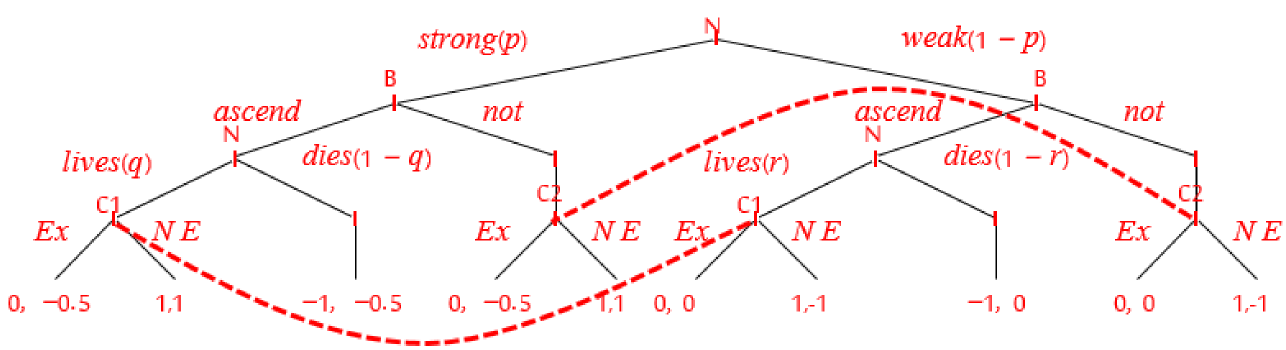
Figure 1: Graph of non-linear fitness advantage $\Delta w = 2(s_A - 0.5)(s_A - 0.75)$, with A referring to technology 1.

climb by Blaine, or a decision by Blaine to not ascend the climb, the clan gets to decide whether to expel or not expel.

The payoffs to Blaine are:		The payoffs to the Clan are:	
Alive, Not Expelled	1	Expel Strong Blaine	-1/2
Alive, Expelled	0	Not Expel Strong Blaine	1
Dead	-1	Expel Weak Blaine	0
		Not Expel Weak Blaine	-1
		Strong Blaine Dies	-1/2
		Weak Blaine Dies	0

1. Draw the extensive form game for this situation. (4 pts)

Figure 2: EFG



2. Find values of the parameters p, q, and r for which the following profile is a Perfect Bayesian Nash Equilibrium (PBE): (8 pts)
   - Strong Blaine Ascends, Weak Blaine chooses Not Ascend.

- Clan Expels Blaine if he or she chooses Not Ascend; Clan chooses to Not Expel Blaine if he or she Ascends without dying.
- Clan believes Blaine is Strong with probability 1 if Ascent chosen, and believes Blaine is Weak with probability 1 if Ascent not chosen.

A separating equilibrium such that strong B always chooses to ascend and weak B always chooses to not, can only exist if expected payoffs are as follows:

(a) $\mathrm{E}u_B$[ascend|strong]$\geq$ $\mathrm{E}u_B$[not|strong], and

(b) $\mathrm{E}u_B$[not|weak]$\geq$$\mathrm{E}u_B$[ascend|weak]

First conditions implies that $(q \times 1 + (1-q) \times -1) \geq 0 \implies q \geq 0.5$.
Second conditions implies that $(2r-1) \leq 0 \implies r \leq 0.5$.
So the separating equilibrium described exists for all p; $r \leq 0.5$; and $q \geq 0.5$

3. For which values of p, q, and r is the following a Perfect Bayesian Nash Equilibrium (PBE):
   - Blaine always chooses to Not Ascend.
   - Clan always chooses to Not Expel Blaine.
   - Clan believes Blaine is Strong with probability p and weak with probability 1-p after Blaine's decision, no matter what he or she chooses.

   (8 pts).

   The Clan's choices have the following payoffs:
   Expel: $p(-1/2) + (1-p)(0)$, and
   Not Expel: $p(1) + (1-p)(-1)$.
   Thus Not Expel is a best response if $2p - 1 \geq -\frac{p}{2} \implies p \geq \frac{2}{5}$.

4. a) Suppose that the Clan leaders want to choose a rock face danger level q that supports the separating PBE described in part b. Assume that the probability a weak climber survives the climb is r = (3/5)q. What value of q maximizes expected Clan payoff in that PBE? (8pts)

   Expected pay-off of the clan based on the separating beliefs in (b):
   p[Strong Blaine will ascend + Chance q s/he will survive clan will not expel him + chance $(1-q)$ s/he will die.] + $(1-p)$[Weak Blaine will not ascend + clan will expel him].
   $= p[q * 1 + (1-q) * (-0.5)] + (1-p) * (0)$
   $= \frac{1}{2}p(1+3q)$

   This is maximized by making q as large as possible, subject to the constraint that the PBE requires $q \geq \frac{1}{2}$ and $r \leq \frac{1}{2}$. We can see that $r \leq \frac{1}{2}$ is binding, so $q = \frac{5}{6}$ maximizes Clan's expected payoff.