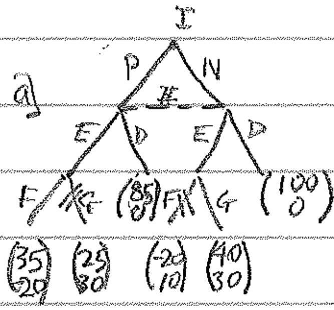
Answer Key: 204B Final Exam. W17

① a]



b] $E \in BR_E \Leftrightarrow p(-20)+(1-p)30 \geq 0$

$\Leftrightarrow 30 \geq 50p \Leftrightarrow p \leq 3/5$.

c] First step in BI shown, Remaining NFG is

	E	D
PF_pG_N	35, −20	85, 0
NF_pG_N	40, 30	100, 0

So SPNE is (NF_pG_N, E) with $p=0$.

That is, it is a (weakly) dominant strategy, and SGP, for I to Not prepare, to Fight if prepared (not on eq. path) and to Go easy in not prep'd, and entrant's BR is to Enter.

② a) Simple BI gives $(Out_{PLCL}, Out_{PHCL}, In_{PHCH}, In_{PLCH})$ for entrant,

thus $(P_H|C_H, P_L|C_L)$, with expected payoffs $(1,1)_{C_H}(.2)+(3,1)_{C_L}(.8)=\binom{2.6}{1}$.

b) Try $(P_H|C_H, P_L|C_L)$, so $\mu(C_H|P_H)=1=\mu(C_L|P_L)$. Then

(*)$[BR_2(P_H)=In, BR_2(P_L)=Out]$. But BR_1 to (*) includes $P_L|C_H$, breaking this eq

Try $(P_L|C_H, P_H|C_L)$ so $\mu(C_H|P_L)=1=\mu(C_L|P_H)$. Then

(**)$[BR_2(P_H)=Out, BR_2(P_L)=In]$. But BR_1 to (**) includes $P_H|C_H$, breaking the eq

Thus neither possible pooling strategy is part of a PBE.

c) Try $(P_H|C_H, C_L)$. So $\mu(C_H|P_H)=.2$ (the prior) and $\mu(C_H|P_L)=q\in[0,1]$ arbitrary.

$BR_2(P_H)=Out$, $BR_2(P_L)=In$ iff $q\geq .5$. Then

$BR_1(C_H)=P_H$ ✓, $BR_1(C_L)=P_H$ if $q\geq .5$ ✓ So a pooling PBE is

$\pi^*=(P_H|C_H,C_L)$ $\mu(\cdot|P_H)=$ prior, $\mu(C_H|P_L)=q\geq 0.5$.

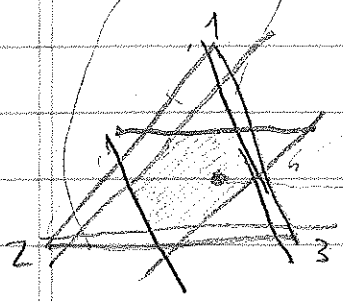
Try $(P_L|C_H,C_L)$. So $\mu(C_H|P_L)=$ prior, $\mu(C_H|P_H)=q\in[0,1]$ arbitrary

$BR_2(P_L)=Out$, $BR_2(P_H)=In$ iff $q\geq 0.5$. Then

$BR_1(C_L)=P_L$ ✓, $BR_1(C_H)=P_L$ iff $q\geq 0.5$. So again we have a pooling PBE

$\pi^*=(P_L|C_H,C_L)$; $\mu(\cdot|P_L)=$ prior, $\mu(C_H|P_H)\geq 0.5$; $a^*(P_L)=Out$, $a^*(P_H)=In$

③ $w(1)=1,\ w(2)=2,\ w(3)=3,\ w(12)=6,\ w(13)=8,\ w(23)=10,\ w(123)=18.$



Core, e.g. $(6,6,6) \in$ Core.

$x_1 \in [1,8]$
$x_2 \in [2,10]$
$x_3 \in [3,12]$

b

ρ	MC_1	MC_2	MC_3
123	1	5	12
132	1	10	7
213	4	2	12
231	8	2	8
312	5	10	3
321	8	7	3
Σ	27	36	45
$\phi =$ SV.	$\boxed{9/2}$	6	$15/2$
normalized	$1/4$	$1/3$	$5/12$

c. Yes, since w is convex (supermodular), $\phi(w) \in \text{Core}(w)$

d. NBS: $\max\limits_{x} (x_1-1)(x_2-2)(x_3-3)$ s.t. $x_1+x_2+x_3=18$

$\Leftrightarrow \max\limits_{y}\ y_1 y_2 y_3$ s.t. $y_1+y_2+y_3 = 18-1-2-3=12$

$\Rightarrow y_i = 4 \Rightarrow \boxed{x_1 = 4+1=5,\ x_2=4+2=6,\ x_3=4+3=7}$

④ a) $W_i = CE_i = M_i + 0.2\,Var_i = \begin{cases} 1+(0.2)1^2 = \underline{1.2}, & i=L \\ 2+(0.2)2^2 = \underline{2.8}, & i=H \end{cases}$

b) $E\,\text{loss} = (.4)2 + (.6)1 = \boxed{1.4\ \text{K/yr} = P}$

c) At $P=1.4$, low risk people refuse $(1.2<1.4)$, so only H-types accept

$E\,\text{profit} = 4000(P - E\,\text{loss}|H) = 4000(1.4-2) = -2400\text{K}$ or $\boxed{\$2.4\text{M loss}}$

d) Assuming a uniform price, insurers will serve only H types (as just seen), at $P = \underset{\text{cost}}{2} + \underset{\text{allowed margin}}{.4} = \underline{2.4\ \text{K/yr}}$.

e) With free entry, P gets bid down to 0-π level, $\underline{P=2}$.

f) Use screening model, find IC's + PC to separate contracts aimed at H, L types.
PC's imply that an upper bound on profit for each H customer is $(0.2)\,2^2 = 0.8$
and is $(0.2)1^2 = 0.2$ for each L-customer, or $(0.2)6000 + (0.8)4000 = 44{,}000\text{K} = \underline{\$44\text{M}}$

g) U is not equivalent to Eu, as explained in the Notes1 p24+.
It is equivalent up to second order. Over a limited range, the function $u(x) = x - cx^2$ works.
See also PS1 problem #2.

5) a) Yes, it is symmetric in that Col's payoff matrix is the transpose of Row's.

b) For $x \in (-2,0)$, we have $p^* = \frac{a_2}{a_1+a_2} = \frac{x}{-2+x} \in (0,1)$, e.g., $p^* = 1/3$ for $x=-1$.

It is a downcrossing since $0 > a_1 = 3-5$, $0 > a_2 = x-0$, hence a unique NE & stable.

c) For $x \in (0,10)$, $a_2 = x > 0 > a_1 = -2$, hence S_2 is a dominant strategy
the Pure NE S_2 is $\therefore$ globally stable.

d) Since $a_1 = -2 < 0$, the CO case with 2 pure NE is not possible.

e) With $x = 1$, (S_2, S_2) is the stage game NE. To sustain cooperation,
consider trigger strategy: play S_1 until someone first plays S_2, then play S_2 ever after.

Playing S_1 (or trigger) against trigger yields stream $3, 3, 3, \ldots$ (*)

S_2 " " " " $5, 1, 1, 1, \ldots$ (**)

∴ * is BR (hence (trigger, trigger) $\in$ NE) iff $PV(*) \geq PV(**) \Leftrightarrow \frac{3}{1-\delta} \geq 4 + \frac{1}{1-\delta}$
$\Leftrightarrow 2 \geq 4(1-\delta) \Leftrightarrow \delta \geq \frac{1}{2}$ $\therefore$

If $\delta = \frac{g}{1+r}$, then the condition is $g \geq \frac{1}{2}(1+r)$.

2. Two firms, Ace and Best, produce brisquets at respective marginal costs $c_A = 4$ and $c_B = 6$. Consumers treat the two brands of brisquets as perfect substitutes.
   a. Suppose that the two firms independently choose output and that inverse demand is $p = 56 - 2Q$, where $Q = q_A + q_B$ is the total output for the two firms. Write down the payoff (i.e., profit) functions for the two firms, find their best responses and the Nash equilibrium outputs and profits. Be sure to mention any other assumptions required to obtain your answer. (10pts)

$C_A = 4$
$C_B = 6$

(a) choose q_A, q_B (cournot)

$P = 56 - 2q_A - 2q_B$

$$\boxed{\Pi_A = (P - C_A)q_A = (56 - 2q_A - 2q_B - C_A)q_A}$$

FOC wrt q_A: $56 - 4q_A - 2q_B - C_A = 0$

$\Rightarrow 4q_A = 56 - 2q_B - C_A$

$\Rightarrow q_A = \dfrac{56 - 2q_B - C_A}{4}$ $\left(= \dfrac{52 - 2q_B}{4} = 13 - \tfrac{1}{2}q_B \right)$

$$\boxed{\Pi_B = (P - C_B)q_B = (56 - 2q_B - 2q_A - C_B)q_B}$$

FOC wrt q_B: $56 - 4q_B - 2q_A - C_B = 0$

$\Rightarrow 4q_B = 56 - 2q_A - C_B$

$\Rightarrow q_B = \dfrac{56 - 2q_A - C_B}{4}$ $\left(= \dfrac{50 - 2q_A}{4} = 12.5 - \tfrac{1}{2}q_A \right)$

SO $\boxed{BR_A(q_B) = 13 - \tfrac{1}{2}q_B}$

$\boxed{BR_B(q_A) = 12.5 - \tfrac{1}{2}q_A}$

solve for q_A^*, q_B^*

$q_A^* = 13 - \tfrac{1}{2}q_B^*$

$q_B^* = 12.5 - \tfrac{1}{2}q_A^*$

$q_B^* = 12.5 - \tfrac{1}{2}(13 - \tfrac{1}{2}q_B^*) = 12.5 - 6.5 + \tfrac{1}{4}q_B^*$

$\Rightarrow \tfrac{3}{4}q_B^* = 6 \Rightarrow \boxed{q_B^* = 8}$ ✓

$q_A^* = 13 - \tfrac{1}{2}(8)$

$= 13 - 4$

$\boxed{q_A^* = 9}$ ✓

Then $P = 56 - 2(9) - 2(8) = 56 - 18 - 16$
$= \underline{22}$

SO $\Pi_A = (22 - 4)(9)$
$= 18 \cdot 9$
$= \boxed{\$162}$ ✓

$\Pi_B = (22 - 6)(8)$
$= 16 \cdot 8$
$= \boxed{\$128}$ ✓

Assuming NO fixed costs! ✓

b. Now suppose that the two firms independently choose price, and the lowest price firm gains the entire market. Assume that this firm sells a quantity consistent with the inverse demand function above. Again find the payoff functions for the two firms, and find their best responses and the Nash equilibrium prices and profits. Be sure to mention any other assumptions required to obtain your answer. (8pts)

b) choose P_A, P_B (Bertrand)

$$\Pi_A = \begin{cases} (P_A - C_A)\left(\frac{56-P_A}{2}\right) & P_A < P_B \\ \frac{1}{2}(P_A - C_A)\left(\frac{56-P_A}{2}\right) & P_A = P_B \\ 0 & P_A > P_B \end{cases}$$

$$\Pi_B = \begin{cases} (P_B - C_B)\left(\frac{56-P_B}{2}\right) & P_B < P_A \\ \frac{1}{2}(P_B - C_B)\left(\frac{56-P_B}{2}\right) & P_B = P_A \\ 0 & P_B > P_A \end{cases}$$

The Best response for each player can't be found from focs (b/c discontinuous), but we know that each player i will try to play $P_i = P_j - \varepsilon$; $\varepsilon > 0$ (arbitrarily small). In this way
if below monopoly price —
i would gain the whole market. This continues until $P_i = C_i$.

In this case since $C_A \neq C_B$, "Best" will not produce below $P_B = 6 = C_B$ (because $\Pi_B = 0$ is preferable to $\Pi_B < 0$). Since "Ace's" $MC = 4 < 6$, he is willing to price $P_A < 6$. Thus in →

equilibrium, we have that:

$$\begin{cases} P_A = 6 - \varepsilon \approx 6 \\ P_B = 6 \end{cases}$$

And A takes the whole market whilst B gets none. Profits are:

$$Q = \frac{56 - P_A}{2} = \frac{56-6}{2} = 25$$

$$\Pi_A = (P_A - C_A)(25) = (6-4)(25) \boxed{= \$50}$$

$$\Pi_B = (P_B - C_B)(0) \boxed{= \$0}$$

Again, assuming no fixed costs. 8/8

which sort of industries is the game in part a more descriptive than the game in part b? Explain very briefly. (2pts)

I guess this would be a better model if a company had capacity concerns/constraints like maybe a factory where you can't choose price b/c you ~~may not be able to produce that quantity.~~

3. An industry consists of a large population of firms, each of which must choose one of two alternative technologies. The first technology has decreasing returns to scale when rare and increasing returns when common; its profitability can be expressed as $u_1 = 2s_1^2 - 2s_1 + 1$, where s_1 is the fraction of industry output produced using that technology. Technology 2 has moderately decreasing returns at all scales; its profitability is $u_2 = 0.5(1.5 - s_2) = 0.5(s_1 + 0.5)$ when fraction $s_2 = 1 - s_1$ of the output is produced using it.

a. Write down the payoff difference $D = u_1 - u_2$ as a function of s_1, and graph this function. (6pts)

Two technologies

$$\begin{cases} \text{tech 1} & u_1 = 2s_1^2 - 2s_1 + 1 \\ \text{tech 2} & u_2 = 0.5(s_1 + 0.5) \end{cases}$$

when fraction s_1 of output is produced using it.

a) $D = u_1 - u_2$

$= 2s_1^2 - 2s_1 + 1 - \left[\frac{1}{2}\left(s_1 + \frac{1}{2}\right)\right]$

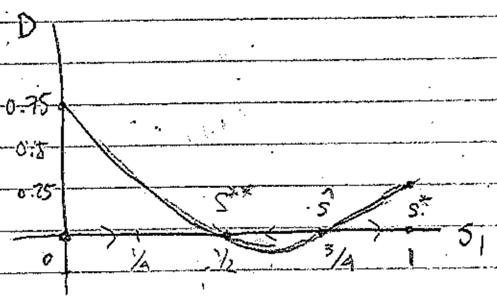
$= 2s_1^2 - 2s_1 + 1 - \frac{1}{2}s_1 - \frac{1}{4}$

$\boxed{D = 2s_1^2 - 2.5s_1 + 0.75}$

$2 - 2.5 + 0.75$

$s_1 = \frac{2.5 \pm \sqrt{6.25 - 6}}{2(2)}$

$s_1 = \frac{2.5 \pm 0.5}{4} = \left(\frac{1}{2}, \frac{3}{4}\right)$



6/6

$4s_1 - 2.5 = 0$

$s_1 = \frac{2.5}{4} = 0.625 \Rightarrow D = 2(0.625)^2 - 2.5(0.625) + 0.75 = -0.03125$

b. Does this game have any pure strategy NE? I.e., is $s_1 = 0$ or 1 a NE? Please verify your answer. (4pts)

b) If $s_1 = 0$, then $D(0) = 0.75 > 0$, means people tend to choose S_1, so not a NE

If $s_1 = 1$, then $D(1) = 0.25 > 0$. means people won't deviate to S_2.

Thus, $s_1 = 1$ is a pure NE.

c. Suppose that sign preserving dynamics describe the evolution of technology adoption in the industry. Find the evolutionary equilibria and their basins of attraction, using the graph from part a. (6pts)

d. Suppose that the second technology is more recent. Predict the long-run shares of the two technologies. (2pts)

Diagram shows that $\hat{s}_1 = 3/4$ separates basins of attraction for $s_1^* = 1$ and $s^{**} = \frac{1}{2}$. the two EE

If 2nd tech. is more recent, then initial state is $s_1 = 1$. So evolutionary (sign-preserving) dynamics never leave its basin $\Rightarrow$ $s_1 = 1$ in the LR.
$s_2 = 0$