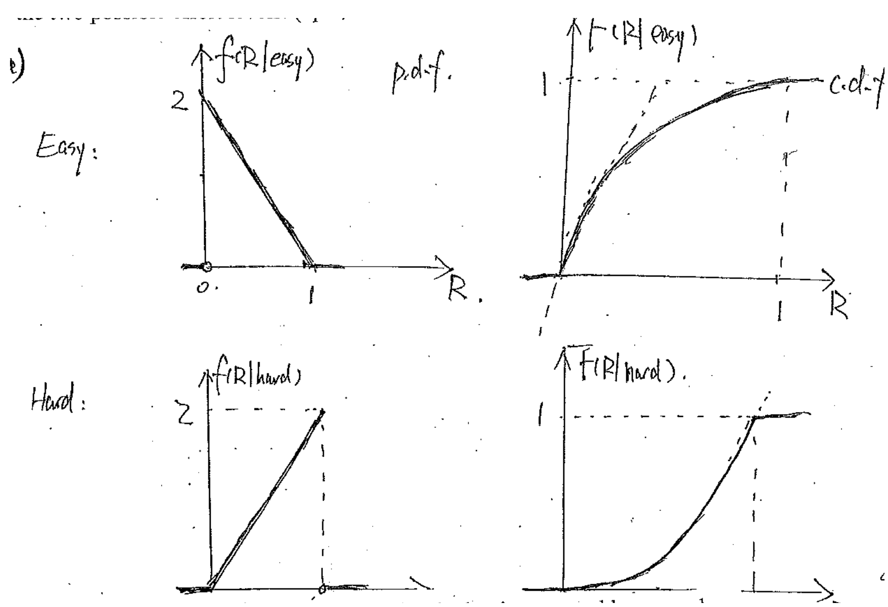
PracProb 1

a. [give standard definitions of these terms.]
b. Use any of these terms to explain Sherlock's insight. One possible approach: We have a separating equilibrium of a signalling game in which the dog is the informed party (Sender), and his message (Bark or Don't Bark) credibly signals to Sherlock (the uninformed party, the Reciever) the true state of Nature, that the thief was a person known to the dog.

PracProb 2



b. The R distribution given hard work FOSD's the distribution for taking it easy, as can be seen from inspecting the figure, or by writing out the cdf's: $F_H(R) = \int_0^R 2xdx = R^2 < 2R - R^2 = \int_0^R 2-2xdx = F_E(R)$ for all $R \in [0,1)$.
Since the two distributions have different means (direct calculation shows $\mu_{r|H} = 2/3, \mu_{r|E} = 1/3$), they can not be ranked by SOSD.
c. $E(u|H) = \int_0^1 u(.25R)f(R|H)dR = \int_0^1 2R^{.5}RdR = 4/5$.
$u(CE|H) = 4/5 \implies CE|H = u^{-1}(.8) = (0.5 * .8)^2 = 0.16$.
$E(u|E) = \int_0^1 u(.25R)f(R|E)dR = \int_0^1 R^{.5}(2-2R)dR = 8/15$.
$u(CE|H) = 8/15 \implies CE|H = u^{-1}(0.5 * 8/15) = (4/15)^2 \approx 0.071$.

PracProb 3

a. $E(U|H) = 4/5 - 0.5 = 0.3$
$E(U|E) = 8/15 - 0 > 0.3$.
Thus taking it easy is his preferred effort choice.
Actually, the incentive constraint $Eu > \bar{u} = 1$ is not met in either case, so the salesman is probably looking for a better job!

b. Assuming that effort level (or other correlates of effort besides R) is not observable, and that the boss is risk neutral, nothing else is required.
c. Note that $u'(w) = w^{-0.5}$, and that $\frac{f(R|E)}{f(R|H)} = \frac{2-2R}{2R} = \frac{1}{R} - 1$.
The formula says, to efficiently motivate high effort, the wage schedule should be

$$\frac{1}{u'(w(R))} = \gamma + \mu \left[1 - \frac{f(R|E)}{f(R|H)}\right] \quad (1)$$

$$w^{0.5} = \gamma + \mu \left[2 - \frac{1}{R}\right] \quad (2)$$

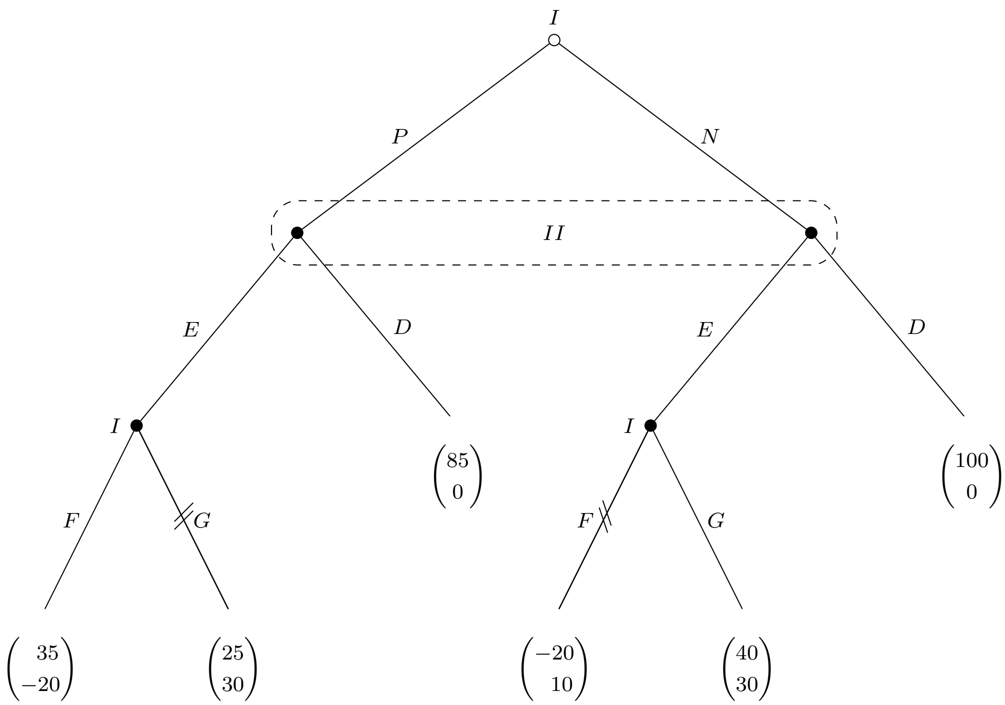
$$w(R) = (\gamma + \mu \left[2 - \frac{1}{R}\right])^2, \quad (3)$$

where γ and μ are chosen to meet the PC and IC.
d. In reality, it is hard to estimate the salesman's true Bernoulli function (assuming that it exists) and the reservation utility $\bar{u}$ could fluctuate as alternative opportunities arise or disappear. The distributions of R might also be wrong. Perhaps more importantly, the salesman should feel that the bonus schedule is fair, and that the penalties for falling below $R = 0.5$ (where he gets only the base salary γ) are not too harsh.
In this particular example, the formula fails (in essence, calls for a negative wage) when the LHS of (2) is negative, i.e., when $R < \frac{\mu}{2\mu - \gamma}$, basically saying the salesman should be fired any time the observed R falls sufficiently low. This is pretty harsh, and would be bad for morale.
e. In view of the low-R issue just mentioned, computing μ and γ seems problematic.

Remaining problems are from the 2017 Final Exam

Problem 1

(a)



Player I is incumbent and Player II is potential entrant.

(b)

$$\begin{aligned} E \in BR_{II} &\Leftrightarrow p(-20) + (1-p)30 \geq 0 \\ &\Leftrightarrow 30 \geq 50p \\ &\Leftrightarrow p \leq \frac{3}{5} \end{aligned}$$

That is, Player II (potential entrant) will choose to enter if $p \leq \frac{3}{5}$.

(c)

The first step of backward induction (BI) is shown in the game tree in part (a). The remaining normal form game (NFG) is

I \ II	E	D
PF_PG_N	$35, -20$	$85, \underline{0}$
NF_PG_N	$\underline{40}, \underline{30}$	$\underline{100}, 0$

So the SPNE is (NF_PG_N, E) with $p = 0$.

That is, for the incumbent, it is a (weakly) dominant strategy and subgame perfect (SGP) to not prepare (N), to fight if prepared (F_P, not on the equilibrium path) and to go easy if not prepared (G_N) and the entrant's best response is to enter (E).

Problem 2

(a)

Simple BI gives $(Out_{P_L C_L}, Out_{P_H C_L}, In_{P_H C_H}, In_{P_L C_H})$ for entrant, thus $(P_H|C_H, P_L|C_L)$ for incumbent, with expected payoffs $(1,1)_{C_H} \cdot (.2) + (3,1)_{C_L} \cdot (.8) = (2.6, 1)$.

(b)

1) Try $(P_H|C_H, P_L|C_L)$.

   So the beliefs can be updated as $\mu(C_H|P_H) = 1$ and $\mu(C_L|P_L) = 1$.

   Then $\{BR_2(P_H) = In, BR_2(P_L) = Out\}^{\cdots(*)}$, but BR_1 to $(*)$ includes $P_L|C_H$, breaking this candidate equilibrium.

2) Try $(P_L|C_H, P_H|C_L)$.

   So the beliefs can be updated as $\mu(C_H|P_L) = 1$ and $\mu(C_L|P_H)$.

   Then $\{BR_2(P_H) = Out, BR_2(P_L) = In\}^{\cdots(**)}$, but BR_1 to $(**)$ includes $P_H|C_H$, breaking this equilibrium.

Thus, neither possible separating strategy is part of a PBE.

(c)

1) Try $(P_H|C_H, C_L)$.

   So the beliefs are $\mu(C_H|P_H) = .2$ (the prior) and $\mu(C_H|P_L) = q \in [0,1]$ (i.e. arbitrary).

   Then, $BR_2(P_H) = Out$ and $BR_2(P_L) = In$ iff $q \geq .5$.

   Then, $BR_1(C_H) = P_H$ and $BR_1(C_L) = P_H$ if $q \geq .5$.

   So a pooling PBE is

$$\big\{m^* = (P_H|C_H, C_L); \mu(\cdot|P_H) = prior, \mu(C_H|P_L) = q \geq 0.5; a^*(P_L) = In, a^*(P_H) = Out\big\}.$$

2) Try $(P_L|C_H, C_L)$.

   So the beliefs are $\mu(C_H|P_L) = prior$ and $\mu(C_H|P_H) = q \in [0,1]$ (i.e. arbitrary).

   Then, $BR_2(P_L) = Out$ and $BR_2(P_H) = In$ iff $q \geq .5$.

   Then, $BR_1(C_L) = P_L$ and $BR_1(C_H) = P_L$ if $q \geq .5$.

   So we have another pooling PBE:

$$\big\{m^* = (P_L|C_H, C_L); \mu(\cdot|P_L) = prior, \mu(C_H|P_H) = q \geq 0.5; a^*(P_L) = Out, a^*(P_H) = In\big\}.$$

Problem 3

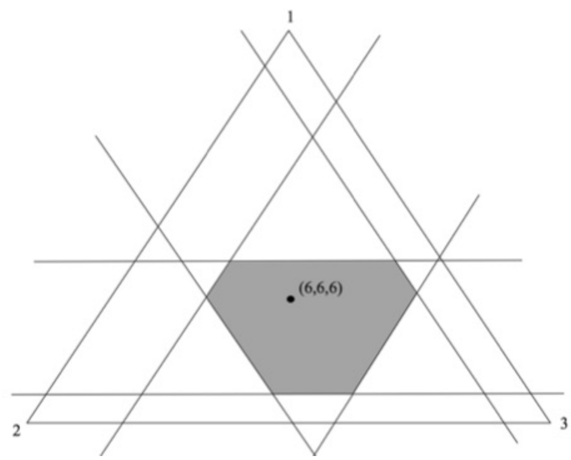
(a)

$$w(\phi) = 0 \qquad w(\{1\}) = 1 \qquad w(\{1,2\}) = 6 \qquad w(\{1,2,3\}) = 18$$
$$w(\{2\}) = 2 \qquad w(\{2,3\}) = 10$$
$$w(\{3\}) = 3 \qquad w(\{1,3\}) = 8$$

The core is sketched below and is defined by the equations

$$x_1 \in [1,8], \quad x_2 \in [2,10], \quad x_3 \in [3,12],$$
$$x_1 + x_2 + x_3 = 18.$$

(Why? e.g., $x_3 < 3$ is blocked by $K = \{3\}$, and $x_3 > 12$ is blocked by $K = \{1,2\}$.)
An example of a core allocation is $(6,6,6)$.



(b)

ρ	MC_1	MC_2	MC_3
123	1	5	12
132	1	10	7
213	4	2	12
231	8	2	8
312	5	10	3
321	8	7	3
$\sum$	27	36	45
ϕ_i	9/2	6	15/2

The normalized Shapley values are $(1/4, 1/3, 5/12)$.

(c)

Yes, $\phi(w) \in Core(w)$ since w is convex (supermodular). Convexity can be seen by inspecting the definition of the ChF: the marginal contribution of i is $|K|i$, which is increasing in the size $|K|$ of the coalition.

(d)

The NBS solves

$$\begin{aligned} &\max_{x_1,x_2,x_3} (x_1-1)(x_2-2)(x_3-3) \quad \text{s.t.} \quad x_1+x_2+x_3=18 \\ \Leftrightarrow &\max_{y_1,y_2,y_3} y_1y_2y_3 \quad \text{s.t.} \quad y_1+y_2+y_3=18-1-2-3=12 \\ \Rightarrow &\quad y_i=4 \quad \text{for } i=1,2,3 \\ \Leftrightarrow &\quad x_1=5, x_2=6, x_3=7 \end{aligned}$$

where we set $y_i = x_i - i$.

Problem 4

(a)

$$w_i = CE_i = \mu_i + 0.2\sigma_i^2 = \begin{cases} 1+0.2\cdot 1^2 = 1.2 & (i=L) \\ 2+0.2\cdot 2^2 = 2.8 & (i=H) \end{cases}$$

(b)

$$E(loss) = (.4)2 + (.6)1 = 1.4 = P$$

(c)

At $P = 1.4$, low risk people refuse $(1.2 < 1.4)$, so only H-type people accept. Then,

$$E(profit) = 4000(P - E(loss|H) = 4000(1.4-2) = -2400,$$

which is \$ 2.4 million loss.

(d)

Assuming a uniform price, insurers will serve only H types (as just seen) at the price $P = 2 + .4 = 2.4$.

(e)

With free entry, P gets down to zero-profit level, so $P = 2$.

(f)

Use screening model and find the insurance company's participation constraint (PC) to separate contracts aimed at H-types and L-types. The PC's imply that an upper bound in profit for each H-type customer is $(0.2)2^2 = 0.8$ and for each L-type customer is $(0.2)1^2 = 0.2$, or $(0.2)6,000 + (0.8)4,000 = 44,000$, which is \$ 44 million.

(g)

U is not equivalent to Eu, as explained in the Notes 1 (p.24+). It is equivalent up to second order. Over a limited range, the function $u(x) = x - cx^2$ works. See also Problem 2 of Problem Set 1.

Problem 5

(a)

Yes, it is symmetric, since the column player's payoff matrix is the transpose of the row player's.

(b)

For $x \in (-2, 0)$, we have $p^* = \frac{a_2}{a_1+a_2} = \frac{x}{-2+x} \in (0, 1)$, e.g., $p^* = 1/3$ for $x = -1$. It is downcrossing since $0 > a_1 = 3 - 5$ and $0 > a_2$, hence a unique, stable NE.

(c)

For $x \in (0, 10)$, $a_2 = x > 0 > a_1 = -2$, hence s_2 is a dominant strategy. Therefore, the pure NE s_2 is globally stable.

(d)

Since $a_1 = -2 < 0$, the CO case with two pure NE is not possible.

(e)

With $x = 1$, (s_2, s_2) is the stage game NE. To sustain cooperation, consider trigger strategy: play s_1 until someone first plays s_2, then play s_2 ever after.
Playing s_1 (or trigger) against trigger yields stream $3, 3, 3, ...$ $(*)$.
Playing s_2 against trigger yields stream $5, 1, 1, ...$ $(**)$.
$(*)$ is BR ($\therefore$ $(trigger, trigger) \in NE$) iff

$$\begin{aligned} PV(*) \geq PV(**) &\Leftrightarrow \frac{3}{1-\delta} \geq 5 + \frac{\delta}{1-\delta} \\ &\Leftrightarrow 3 \geq 5(1-\delta) + \delta \\ &\Leftrightarrow \delta \geq \frac{1}{2}. \end{aligned}$$

If $\delta = \frac{q}{1+r}$, the the condition is $q \geq \frac{1+r}{2}$.