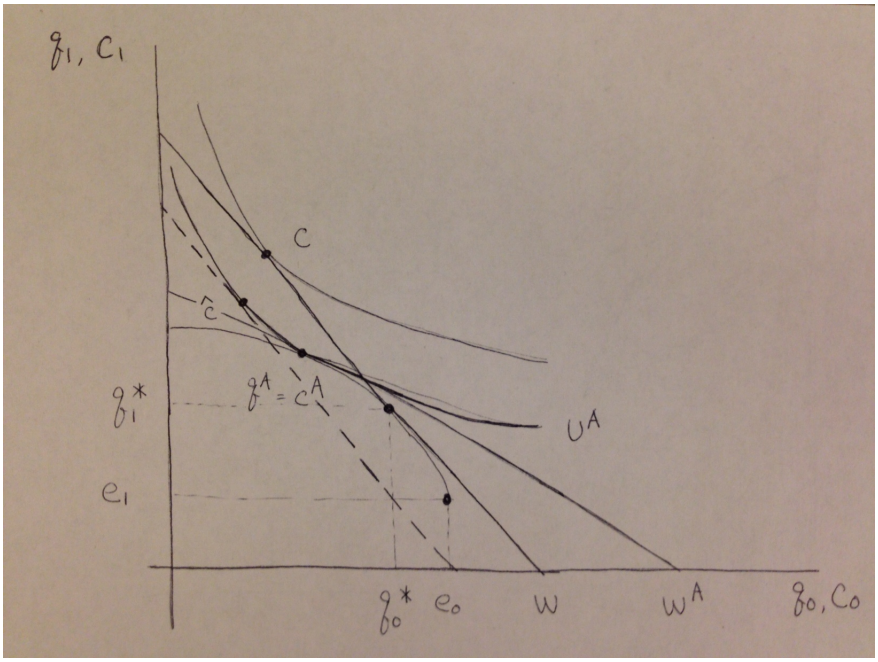
Practice Problems on Choice Over Time

Econ 200

1. North Sanistan is a small isolated country with low productivity and average thrift compared to the rest of the world. Predict the impact on her real interest rate, investment, savings, current and future consumption, and real wealth if North Sanistan (NS) unifies her financial market with the rest of the world.

Solution: In autarky (i.e., before unification), the representative agent in NS is neither a borrower nor lender, so optimal production Q^A equals optimal consumption C^A where the PPF is tangent to an indifference curve u^A. The tangent line here has slope $1 + r^A$, where r^A is the real interest rate pre-unification.
Post unification, the real interest rate is $r > r^A$. Why? Because NS is small, unification will have little impact on the world's real interest rate, and because NS has low productivity r^A is relatively low.
NS's new production point Q is clockwise from Q^A along the PPF; this follows from $f'' < 0$ (i.e., the PPF is concave) and $r > r^A$. That in itself implies less borrowing/more lending via the production effect.
The substitution effect can be found by holding utility constant at u^A. Since preferences are convex and $r > r^A$, this effect moves the income-compensated consumption clockwise on the indifference curve, again implying less borrowing/more lending. Finally, assuming that consumption overall is a normal good in both periods, the income effect (in the diagram, the move from the dotted budget line to the solid line) increases consumption in both periods, implying yet more lending.
Conclusions: Relative to autarky,

- real interest rate goes up, as noted already;
- investment goes down (via clockwise move on PPF);
- savings and lending go up (via both clockwise moves);
- utility and future consumption both increase (by income and sub effects)
- current consumption is ambiguous (decreased by sub effect, probably increased by income effect)
- real wealth is ambiguous (the increase in real interest rate partially or perhaps completely offsets the increase in current and future consumption quantities; see diagram comparing intercepts).



2. Long ago a large kingdom had a real interest rate of about 6% and an inflation rate of about 4% annually for many decades. Then the Master of the Mint started adding base metal to the kingdom's coins, and the annual inflation rose to 24%. Estimate the nominal interest rate before and after the this change in policy.

Solution: By Fisher's equation, $k = r + \pi + r\pi$, where k is nominal interest rate, r is real interest rate, and π is inflation rate.
Before the change, $r = 0.06$, $\pi = 0.04$. Therefore, nominal interest rate is,

$$k = 0.06 + 0.04 + (0.06)(0.04) = 0.1024 = 10.24\%. \tag{1}$$

After the change, $r = 0.06$, $\pi = 0.24$. Therefore, nominal interest rate is,

$$k = 0.06 + 0.24 + (0.06)(0.24) = 0.3144 = 31.44\%. \tag{2}$$

3. Opening a factory will cost 10 up front (all amounts in \$millions) but will yield a net cash flow of 3 per year for 5 years, starting in year 1. In year 5, the factory will have salvage value 2. If the appropriate interest rate is $k = 0.08$, estimate the asset value of the factory. How does your estimate change if, other things equal, the interest rate goes to $k = 0.1$?

Solution: Asset value of the factory given nominal interest rate, k, is,

$$PV_k = -10 + \frac{3}{1+k} + \frac{3}{(1+k)^2} + \frac{3}{(1+k)^3} + \frac{3}{(1+k)^4} + \frac{3+2}{(1+k)^5}. \tag{3}$$

When $k = 0.08$, the asset value is 3.34. When $k = 0.1$, the asset value is 2.61. Since all the cost comes up front, when the interest rate increases, the asset value decreases.