Contents

Chapter 9: Imperfect Competition

A firm has market power when it can influence the market price. The extreme case is monopoly, where the power is unilateral. One consequence is, as we will see, a "deadweight loss" of total surplus.

We then use game theory to present the two classic (static) models of imperfect competition, due to Bertrand and Cournot. Under Bertrand competition, firms simultaneously choose price to maximize profit, while under Cournot competition they simultaneously choose quantity. The two otherwise identical game-theoretic models reach very different conclusions, and we will consider ways to reconcile them.

Other topics in oligopoly are treated even more briefly. We briefly mention product differentiation, sketch tacit collusion in ongoing Bertrand competition, and consider competition with spatially differentiated goods.

1 Market power

Let demand $x(p)$ be a smooth strictly decreasing function.

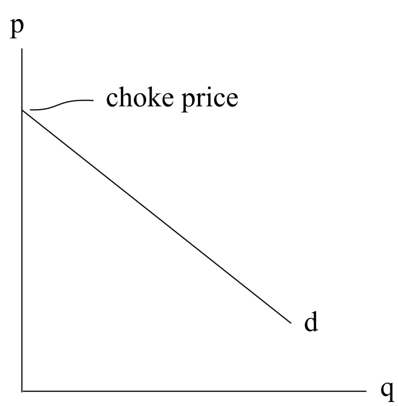

Figure 1: Inverse demand.

Inverse demand $p(.) = x^{-1}(.)$, aka willingness to pay (WTP) or demand price, satisfies the usual assumptions:

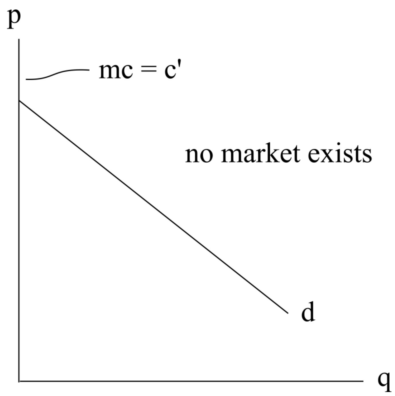

Figure 2: Industry not viable if MC always exceeds WTP.

- $p(q)$ twice differentiable,
- $p'(q) < 0$, and
- $p(0) < \infty$ (choke price exists).

Costs are such that:

- $c(q)$ is twice differentiable,
- $c' > 0$, and
- $c''(q) \geq 0$, so marginal cost is positive and non-decreasing.

Also assume

- $0 \leq c'(0) \leq p(0)$, that is, the industry may be viable.
- $c'(\infty) \geq p(\infty)$, that is, marginal costs eventually rise above WTP.

Then:

1. $\exists\ q_o$ such that

$$p(q_o) = c'(q_o), \tag{1}$$

- i.e. there is a social optimum.
- Proof: Define marginal surplus $z(q) = p(q) - c'(q)$. Current assumptions imply: $z' < 0, z(0) > 0, z(\infty) < 0$.

  Hence the intermediate value theorem tells us that there is a unique q_o such that $0 = z(q_o) = p(q_o) - c'(q_o)$
- Total surplus is $S(q) = \int_0^q z(u)du$. Use the Fundamental Theorem of Calculus to see that it is maximized at q_o.

2. The standard monopolist problem

$$\max_{q \geq 0} \pi = qp(q) - c(q) \tag{2}$$

has solution q_m.

- $0 < q_m < q_o$, as we will see by comparing the first-order conditions.
- The monopolist's FOC is

$$q_m p'(q_m) + p(q_m) \leq c'(q_m), \text{ with equality if } q_m > 0. \tag{3}$$

  The first term is the price effect, which is negative since $p'(q) < 0$, and the second term is the quantity effect. Marginal revenue is the sum of these two effects.
- Equation (3) is the same as (1) except for the negative first term.
- Thus when $q > 0$ we can write q_m as the solution to $z(q) = -qp'(q) > 0$.
- Again applying the intermediate value theorem to z, we conclude that indeed $q_m < q_o$
- The deadweight loss (DWL) due to monopoly is found by integrating the area between demand and marginal cost between q_m and q_o.

$$DWL = \int_{q_m}^{q_o} z(q)dq > 0 \tag{4}$$

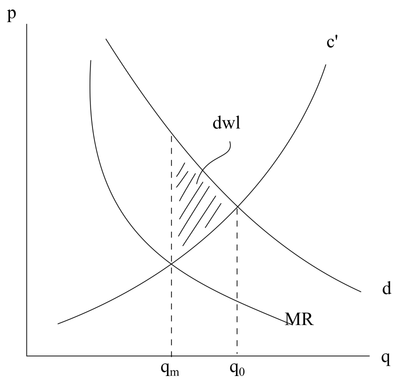

Figure 3: DWL > 0 is the area between q_m and q_o, above marginal cost c' and below inverse demand $p(q)$. It is the surplus on units produced in the social optimum but not in monopoly.

- With perfect price discrimination the profit maximizing output is q_o, so in this case $DWL = CS = 0$.

2 Static Bertrand (1883) model

- Consider a duopoly with two firms j and k. Market demand $x(p)$ satisfies the previous assumptions.
- Production function is the same for both firms and is linearly homogenous. That is, we have constant returns to scale (CRS), which implies constant marginal cost equal to average cost in the long-run: $AC = MC = c$.
- Assume $x(c) \in (0, \infty)$ so that the industry is viable.
- Output is homogenous (i.e., identical goods i.e., no product differentiation).
- Consider the following simultaneous move game: firms announce price p_i and p_j.

- Sales are

$$x_j(p_j, p_i) = \begin{cases} x(p_j) & \text{if } p_j < p_k \\ \frac{1}{2}x(p_j) & \text{if } p_j = p_k \\ 0 & \text{if } p_j > p_k. \end{cases} \tag{5}$$

  - This means we have a winner-take-all market where the firm with the lower price gets all the sales. They split the market if prices are equal.

- Another key assumption is that production costs are only incurred for actual sales (i.e. firms produce to order).

  - Profit is then:

$$\pi_j(p_j, p_k) = x_j(p_j, p_k) \left[p_j - c\right] \tag{6}$$

- Problem: x_j is discontinuous at $p_j = p_k$, so the FOC does not much help.

- Since this is a simultaneous move game, we can look for the Bertrand Nash equilibrium as a simultaneous best-response.

- The best-response functions are in price space since these are the choice variables.

- To avoid technicalities, we will treat price space as discrete with finely spaced grid points, e.g., prices are to nearest penny (0.01).

-
  - Suppose $c = 1$. If $p_j = 6$, the best response by k is $p_k = 5.99$.
  - With p_j on the horizontal axis, the best-response function for k lies just below the 45° line ($p_j = p_k$) for all $p_j > c$.
  - If $p_j \leq c$ then a selection from k's BR correspondence is $B_k(p_j) = c$.
  - Similarly, for player j the best-response function lies just above the the 45° line.

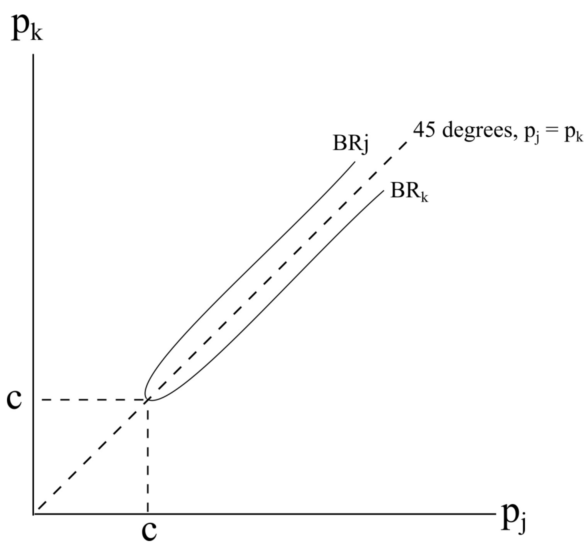

Figure 4: Bertrand Nash equilibrium and best-response functions.

- Up to the nearest penny, we have a unique NE profile and NE payoffs:

$$p_j = p_k = c, \tag{7}$$

$$\pi_j = \pi_k = 0 \tag{8}$$

- Note that $q_o = x(c)$, so...
- with only two firms we obtain the perfectly competitive outcome in this winner-take-all, produce to order framework.
- This same NE is obtained if there are more than 2 identical firms.
- Suppose firm k has a higher marginal cost: $c_k > c_j + 2\epsilon$.
  * The best-response of firm j is to charge epsilon less than c_k and take the entire market, to obtain profit:

$$\pi_j = x\left(c_k - \epsilon\right)\left[c_k - \epsilon - c_j\right] \approx (c_k - c_j)x(c_k) \tag{9}$$

  * An example. Let $P = 100 - Q$ be market demand, with marginal costs $c_k = 12 > c_j = 10$ and $\epsilon = 1$.

  - Then $\pi = \frac{1}{2}(88)(2) = 88$ if $p_j = p_k = c_k$, and $\pi = 89(1) = 89$ if $p_j = c_k - \epsilon$.

Now let's relax the assumption of homogenous goods and look at product differentiation. Now consumers will switch to their less preferred variety only if the price difference is large enough.

Given the pitiful Bertrand-NE profits with homogeneous goods, firms have an incentive to differentiate their products; as you will see, that will raise profits.

Example: linear demand system.

- Begin with linear inverse demands:

$$p_1 = \alpha_1 - \beta_1 y_1 - \gamma y_2 \tag{10}$$

$$p_2 = \alpha_2 - \beta_2 y_2 - \gamma y_1 \tag{11}$$

- the coefficient γ must be the same in both equations from the duality theory you learned last quarter (mixed second partials ...).

- An index of the degree of differentiation is $\sigma = \frac{\gamma^2}{\beta_1 \beta_2} \in [0, 1]$. We have perfect substitutes if $\sigma = 1$, and no substitution if $\sigma = 0$.

- Solve the pair of equations for y_1, y_2 to get direct demands for the Bertrand profit functions.

- Next obtain the BR functions by taking FOC's.

- Both BR functions have *positive slope*: $\frac{\partial BR_k}{\partial p_j}, \frac{\partial BR_j}{\partial p_k} > 0$ for $p_i > c$, $i = j, k$. This property is called "strategic complements." Here it means that when firm k raises price, firm j has an incentive to increase its own price.

- Check that NE profits decrease in $\sigma \in [0, 1]$.

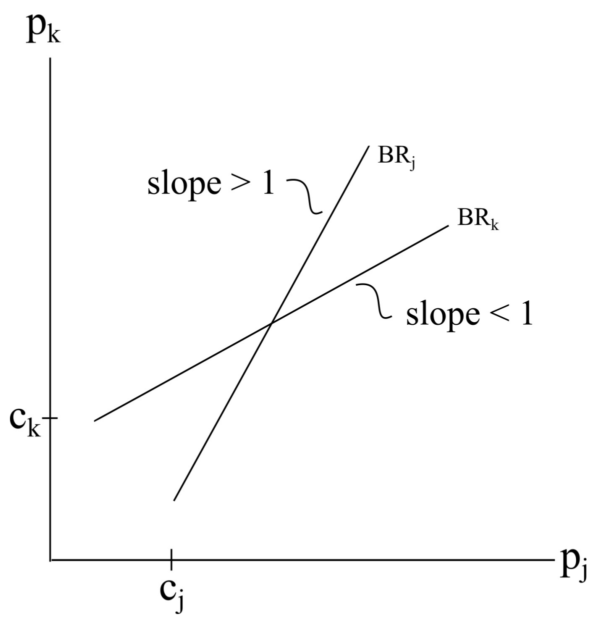

Figure 5: Differentiated products Bertrand Nash equilibrium and best-response functions. Note that $0 < \frac{\partial BR_k}{\partial p_j} < 1$, so the slopes are positive but (viewed from the appropriate axis) less than 1.0.

The later section on the Hotelling spatial model in essence takes σ as a choice variable. The differentiated good Bertrand model is a workhorse in applications, so it is worthwhile working out the details of the above example.

Another way to describe differentiated goods is via demand functions like $y_i(p_i, P) = f(\frac{p_i}{P})$, where P is a suitably defined price index looking at all substitute goods and f is a decreasing function. This approach, developed by Dixit and Stiglitz (19xx) is used in monopolistic competition models, a form of imperfect competition that we will neglect here despite its role in I/O theory and macroeconomics.

3 Bertrand forever

- Let's return to the identical good, equal marginal cost setup. In the 1-shot NE, price is equal to marginal cost and economic profit is zero.

- Consider the infinite horizon ($T = \infty$) repeated Bertrand game and the strategy

$$s_j = \begin{cases} \text{set } p_{j,0} = p_m, \\ \text{set } p_{j,t} = p_m \text{ if } p_{k,t-1} \geq p_m \\ \text{set } p_{j,t} = c \text{ otherwise.} \end{cases} \tag{12}$$

- This is a Bertrand trigger strategy. If it is a best-response to itself, then we have a model of tacit collusion.
  - No communication need take place, but the monopoly outcome is obtained.
  - Any deviation triggers an immediate price war.
  - Note that the 1-shot NE can replace c in a more complicated setup.
- So when is this strategy a BR to itself? Let's find conditions on the discount factor d that support the collusive outcome ($p_j = p_m, p_k = p_m$) as an NE of the repeated game.
- Write out the payoff streams.
  - For example, suppose

$$\begin{aligned} \pi_j(p_m, p_m) &= 2 = \pi_k(p_m, p_m) \\ \pi_j(c, c) &= 0 = \pi_k(c, c). \end{aligned} \tag{13}$$

  - Also, suppose that a player would earn 4 if they chose $p_m - \epsilon$ and the other player chose p_m (if $p_m - \epsilon$ is the 1-shot best-response to p_m that captures the entire market)
  - If both players choose the trigger strategy in (12) then the present value of the payoff stream is

$$PV_d(2, 2d, 2d^2, ...) = 2\sum_{t=0}^{\infty} d^t = \frac{2}{1-d}. \tag{14}$$

– If a player were to deviate then present value of the payoff stream is

$$PV_d(4, 0d, 0d^2, ...) = 4. \tag{15}$$

– Thus tacit collusion is sustainable if

$$\begin{aligned} \frac{2}{1-d} &> 4 \\ 2 &> 4(1-d) \\ d &> \frac{1}{2} \end{aligned} \tag{16}$$

or in terms of the critical discount rate, assuming that the continuation probability is $q = 1$,

$$\begin{aligned} d &\equiv \frac{1}{1+r} > \frac{1}{2} \\ r &< 1. \end{aligned}$$

4 Static Cournot (1838) model

- The economic environment is the same as in Bertrand, but now we will assume that firms simultaneously choose quantity instead of price.

  – MCWG example: farmers pick a perishable crop, take it to market where (via Walrasian auctioneer) the market clearing price is given by demand: $P(q_j + q_k)$.

- Since quantities are chosen, the production costs are incurred on those units, so we drop the "produce to order" assumption used in Bertrand.

- Firms maximize profit, taking the other firms' output as given or, as we shall later say, they conjecture that $\frac{\partial q_k}{\partial q_j} = \frac{\partial q_j}{\partial q_k} = 0$. Another way to think about this is that the market price is not in the firms information set when they are choosing quantity. However, firms do know the cost when they make their quantity choice. By contrast, under Bertrand firms' do not know their quantity and hence realized costs when choosing their price since their quantity depends on the other firm's price.

- Thus the objective of firm j is

$$\max_{q_j \geq 0} P(q_j + q_k)q_j - cq_j \tag{17}$$

- with first-order condition

$$P'(q_j + q_k)q_j + P(q_j + q_k) = c \tag{18}$$

- Implicitly differentiating (18) wrt q_k, you can see that the BR functions are typically[1] negatively sloped, a property known as "strategic substitutes."

- Implicit in Equation (18) is that the firm recognizes that $\frac{\partial P}{\partial q_j} \neq 0$, but assumes $\frac{\partial q_k}{\partial q_j} = 0$.

  – That is, they recognize that changes in their own output will change market price, but not that changes in their output will induce the other firm to change output.

  – Since the BR functions are negatively sloped, $\frac{\partial q_j^{br}}{\partial q_k} < 0$ and $\frac{\partial q_k^{br}}{\partial q_j} < 0$, the firm overestimates the fall in price from an increase in their own output due to the price effect: $P'(q_j + q_k)q_j$.

[1]If demand is linear, so that $P'' = 0$, then this exercise shows that the BR functions have constant slope -0.5.

– Note that the firm's conjecture $\frac{\partial q_k}{\partial q_j} = 0$ is inconsistent with the reaction function slope, a point to which we will return in the next section.

- The Cournot Nash equilibrium can be seen as the intersection of the two firms' BR's, as in Figure 6

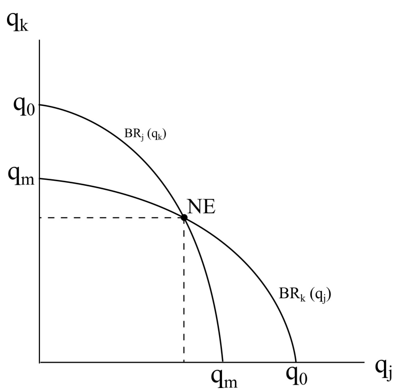

Figure 6: Best response functions and Cournot Nash equilibrium.

Recall the marginal surplus function $z(q)$ from the previous section.

- Using current assumptions and notation (e.g., MC $= c$), it is $z(q) = P(q) - c$, which is a strictly decreasing function since $P' < 0$.

- In symmetric duopoly, equation (18) can be written as $z(q) + \frac{q}{2}P'(q) = 0$, since the firms choose equal quantities, $q_k = q_j = \frac{q}{2}$.

- Figure 7 shows that the this implies that the Cournot Nash equilibrium total quantity q_2 in duopoly lies between q_m and q_o.

- The same reasoning shows that in symmetric Cournot oligopoly with $n > 2$ firms, the Cournot Nash equilibrium total quantity q_n is the solution to $z(q) = -\frac{q}{n}P'(q) > 0$.

- It follows that $q_n \to q_o$ as $n \to \infty$, as indicated in the Figure.

- Thus the limiting case of Cournot competition as the number of firms increases is perfect competition.

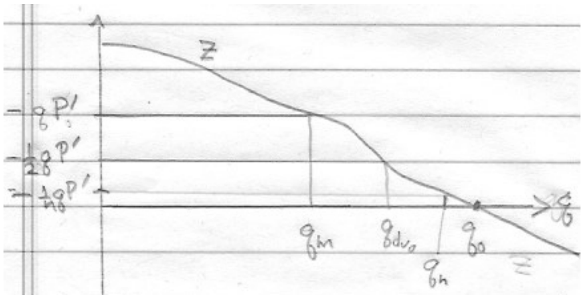

Figure 7: Marginal surplus $z(q) = 0$ at the competitive (or socially optimal quantity q_o. It is $-qP'(q) > 0$ at the monopoly output q_m, and is $-\frac{q}{2}P'(q) > 0$ for a n-firm symmetric Cournot oligopoly.

5 Reconciling Cournot and Bertrand

- Bertrand would seem the better assumption.
  - Most firms choose price (direct approach to model evaluation: validity of the assumptions), but gets the wrong result.
- Cournot is opposite.
  - Cournot gets the right answer: $\pi > 0$, but for the wrong reason (indirect approach: valid conclusions).
- How can we reconcile this apparent disconnect?
  - Think of quantity as a long-run choice of capacity and price as a short-run competition, given these capacity choices.
- If we have Bertrand competition with capacity constraints that are common knowledge then: $p_j = p_k = c$, $\pi_j = \pi_k = 0$ is not a NE.

– If $p_k > c$ then firm j can not supply the entire market.

– This implies $\pi_k > 0$. This was noted by Edgeworth in 1897.

- Kreps and Scheinkman (1983) *Bell Journal of Economics* (now *RAND Journal*) show that under certain conditions (namely that high value demands get satisfied first when low price firm has demand greater than its capacity) that the unique SPNE of the price-competition game with capacity constraints is the Cournot outcome.

6 Conjectural Variations

Recall that, for homogeneous goods with inverse demand $p(Y) = p(y_1 + y_{-1})$, firm 1's problem can be written as $\max_{[y_1 \geq 0]} y_1 p(y_1 + y_{-1}) - c(y_1)$. The first order condition fully written out is

$$c'(y_1) = p(Y) + y_1 p'(Y)[\frac{dy_1}{dy_1} + \frac{dy_{-1}}{dy_1}] \quad (19)$$

$$= p(Y) + y_1 p'(Y)[1 + \nu], \quad (20)$$

where $\nu = \frac{dy_{-1}}{dy_1}$ is firm 1's *conjectural variation* — her belief about how a change in her output y_1 will affect the total output y_{-1} of all rivals.

- $\nu = 0$ is the Cournot conjectural variation. She takes as given her rivals' output level, and (incorrectly!) assumes that she can't affect it.

- What if one firm can commit to a particular output level and (correctly) anticipates that other firms will best respond to it? Such a firm is called a Stackelberg leader, and the others are called followers.

- Stackelberg followers correctly set $\nu = 0$ when thinking about the leader, who is committed and won't respond to followers' quantity choices.

- $\nu = -0.5$ is the Stackelberg leader's conjectural variation in the simple linear duopoly. More generally, it is the slope of other firms' summed reaction functions.

- $\nu = -1.0$ is the competitive or Bertrand conjectural variation. It ensures that price = MC, and says that the firm believes that other firms will replace any units it withholds from the market.

- $\nu = y_{-1}/y_1$ is the collusion conjectural variation – other firms will maintain their current share.

- "consistent" conjectural variations equate ν to the actual comparative statics of the model for each firm (Bresnahan, 1981).

Economic theorists no longer find it fashionable to write down arbitrary expressions for ν, and the idea of consistent conjectures never got much empirical support. But it might be a helpful way to think about applications. See for example the McGinty (2016) treatment of greenhouse gas abatement treaties.

7 Spatial Competition: Hotelling location models.

Let us now take a deeper look at imperfect substitutes. So far, we have taken as given substitution elasticities in utility functions and demand functions. We also noted (from the differentiated good Bertrand model) that firms tend to be more profitable when their products are less substitutable. How can we model that dimension of competition?

Hotelling (1929) apparently was the first to take up that challenge. His "Main Street" model used a spatial metaphor to describe the substitutability among products. Think of the producers choosing the products' characteristics (e.g., fuel economy, acceleration, and seating capacity of cars) in order to fill niches of the market that are relatively undersupplied. That is, firms choose location in the space of characteristics.

Hotelling considered a very special characteristic space: location within the interval $[0, 1]$. This could be taken literally as an address on a small town's Main Street, or metaphorically as in characteristic space.

We begin with a duopoly where firms choose location but not price; for simplicity we assume that price is fixed at $p_1 = p_2 = p > c$, where $c = c_1 = c_2$ is the constant marginal cost faced by both firms.

- Label the firms so that the location choices satisfy $z_1 \le z_2 \in [0, 1]$.

- For simplicity, assume that consumers' preferred locations are uniformly distributed along $[0, 1]$.

- Also, for simplicity, assume linear transportation (or transformation) cost $t > 0$. Thus the delivered price at location z for firm j is $p_j(z) = p + t|z - z_j|$. The assumption is that consumers buy at the lowest delivered price.

- Under current simplifications, this means that firm 1 gets all customers in $[0, \hat{z})$ and firm 2 gets those in $(\hat{z}, 1]$, where $\hat{z}$ solves $p_1(z) = p_2(z)$. In other words, the market shares are $\hat{z}$ and $1 - \hat{z}$, where the customer at $\hat{z} = 0.5(z_1 + z_2)$ faces the same delivered price from both firms.

- Since $p > c$, firms maximize profit by maximizing market share.

- What is firm i's BR to location choice z_j of firm j? If $z_j < 0.5$, it is to locate a tiny bit to the right, at $z_j + \epsilon$. If $z_j > 0.5$, it is to locate a tiny bit to the left, at $z_j - \epsilon$. This is how i maximizes market share.

- So the unique NE of this Hotelling location game is for both firms to locate back-to-back at $z = 0.5$.

This simple game is sometimes used to explain (in part) why firms in a similar line of

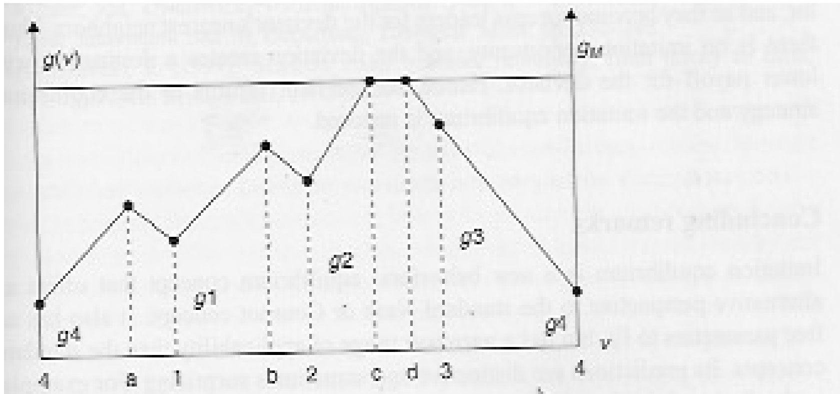

Figure 13.2 Graph showing effective price $g(v)$ as a function of location v. Firm 1 serves territory [a,b], firm 2 serves [b,c], no firm serves (c,d), firm 3 splits [d,3] with firm 4, which serves exclusively the remaining territory.

Figure 8: From Selten chapter in Friedman and Cassar (2004). Here $gi = p_i(z) - c$, and i is location z_i (denoted v in the Figure).

business tend to locate next door to eachother, and why political parties used to adopt very similar platforms.

There are many, many extensions of the model. Expanding the duopoly location problem above to triopoly yields no NE in pure strategies. With 4 firms, the pure strategy NE all have 2 firms back to back at $z = .25$ and the other 2 at $z = .75$.

What if firms first commit to specific locations and then pick prices? Figure 8 illustrates how the shares are determined from an arbitrary set of locations and prices for 4 firms. The analysis is a bit tricky because the endpoints of the line segment play a special role. To make the location space more homogeneous (the technical word is 'isotropic'), one can join the endpoints to make a circle, and this is assumed in the Figure. It can be shown in this case that the unique NE in pure strategies is for firms to space themselves equally around the circle (maximum differentiation) and to all charge the price $p_i = c + t$.

Other variants of the Hotelling location game consider two dimensional locations, on a rectangle or (to make it isotropic) a torus and various numbers of firms. One can also consider nonlinear transportation (or transformation) costs. There seems to be room for applied work here, but I've not seen much published recently. There are recent laboratory experiments by UCSC PhD Curtis Kephart.

8 Behavioral considerations

To be added later. May include

- highlights from Ch 10 of Friedman-Cassar 2004 book,
- whatever matt wants to say about consistent conjectures experiments
- hotelling lab experiments.

9 Further Reading

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