Contents

Chapter 10: Asymmetric Information and Adverse Selection

This chapter considers asymmetric information between players. The seller of a good may have more information about the quality than a potential buyer; for example, buyers cannot detect the quality until after the good has been purchased.

Suppose there is a range of goods types, representing quality or risk. With perfect information price has a single role: to clear markets. With asymmetric information price now has an additional role — it may convey information about the quality.

We present Akerlof's classic "Lemons" model where asymmetric information can lead to a partial or even complete market failure. A competitive equilibrium results in a "shrinking pool" with adverse selection where sellers with high quality types leave the market as the price falls. In the resulting equilibrium only the lowest quality cars are sold and the competitive equilibrium is not socially optimal.

[This is a short chapter and may be combined with the following one if more is not added]

1 Prologue: The takeover game

Consider the two player game in which a conglomerate firm (Player A for Acquirer) buys startup firms (Player T for Target).

- A has expertise in developing startup firms. It is able to increase their value by 50% after acquisition.
- A knows that the (pre-increase) value of potential targets is uniformly distributed on $[0, 100]$. That is all that A knows about T.
- T knows its own value $v \in [0, 100]$ but can't credibly signal it.
- A moves first by bidding $b \in [0, 100]$ to acquire T.

- Then B ends the game by either accepting or rejecting the bid.
- Payoffs are 0 to both players if the bid is rejected.
- If the bid is accepted, then the payoff to B is $b - v$ and the payoff to A is $1.5v - b$.

This game has mutual gains. When the bid is accepted, the payoff sum is $0.5v$, with expected value 25.

Note that in this game B is the informed player (knows v) and A is uninformed (knows only the distribution of v). There is adverse selection to the extent that the distribution of firms that accept bids is less favorable to A than the overall distribution.

As a class demonstration, students play the role of firm A. We can automate B by playing its dominant strategy mechanically: accept $b > v$, reject $b < v$. (Of course, in the zero probability event that $b = v$, either action is a BR.) Dice rolls (with 10-sided dice) determine the realized value of the target each period.

In the last section we will return to this game.

2 Asymmetric information

We now see that asymmetric information can lead to a market failure.

- One of the assumptions of perfect competition that the welfare theorems are derived from is perfect information.
- We will show what happens when we relax this assumption in isolation, otherwise maintaining competitive markets.
- Examples of asymmetric information include:
    1. The seller of a used car knows more about the car than a potential buyer. The car is an "experience good". Experience with the good reveals its type, since a buyer cannot tell the type just by looking at it.

2. A tomato is also an experience good. You don't know how much flavor it has until after you have purchased it.
3. Insurance companies may know less about your individual risk than you do. They can not observe your individual driving habits, so they categorize (and charge a price) based on observable characteristics such as: age, sex, zip code, marital status, income, accident history, etc.
4. Firms do not know the ability or motivation of a potential worker.

- The key: with asymmetric information price does "double duty".
  - Price clears markets, but price that a seller is willing to sell at also reveals something about the "type" of the good.
  - Type may be interpreted as quality.
- If the seller is willing to sell at a low price, then the buyer may think that this price signals that the quality is low, and may be less willing to buy at a lower price.
  - Example: Suppose someone with inside information on a firm offers to sell you his shares at a low price. Should you accept the offer? Probably not.
  - Economists refer to this result as the "Graucho Marx Theorem" or "no trade theorem."They are inspired by Groucho's comment, "I wouldn't want to belong to any club that would have me as a member."

3 Akerlof (1970) "Lemons" Model

- Quality uncertainty and the market mechanism.
- A bad used car is called a lemon.
- Asymmetric information: seller knows quality (θ) of the car, buyers do not.

- θ is a "type" meaning the quality of the car, which we can think of as the value to a buyer. $\theta \in [\theta_1, \theta_2]$, the range of types is common knowledge, but the type of any individual car is not observed by a potential buyer.
- $\Theta(p)$ (for sellers) is the set of cars that are for sale at a given price.
- Suppose θ is uniformly distributed on the interval $2,000 to $3,000, so $\theta_1 = 2.0$, $\theta_2 = 3.0$, and $\theta \sim U[2.0, 3.0]$ in thousands of dollars.

- Seller side of the market.
  - Sellers have a reservation value $r(\theta) = \theta - 0.1$.
    * Thus, there is a surplus of $100 per car since the value to buyers exceeds the value to sellers by 0.1 measured in thousands.
- With perfect information (i.e. no asymmetric information) the equilibrium is that all cars are sold at a price determined by the relative bargaining power of buyers and sellers such that: $p(\theta) \in [\theta - 0.1, \theta]$.
- Now, let's see what happens with asymmetric information.
  - First, let's find $\Theta(p)$, the set of car qualities available at given price p.

$$\Theta(p) = \{\theta : r(\theta) \leq p\} = [2.0, p + 0.1] \tag{1}$$

  - The cars for sale are those whose reservation value is less than or equal to the price. Since $r(\theta) = \theta - 0.1$, $r(\theta) \leq p$ implies $\theta \leq p + 0.1$.
- Buyer side of the market.
  - For simplicity assume that buyers are risk neutral.
  - Buyers' demand is a function of price and beliefs, μ, about the average quality of cars that are for sale.

– Since quality is unobservable buyers form beliefs by taking an expectation of quality.

- Think of a competitive market where demand is

$$D(p) = \begin{cases} 0 \text{ if } \mu < p \\ [0, \infty] \text{ if } \mu = p \\ \infty \text{ if } \mu > p. \end{cases} \tag{2}$$

- Demand is perfectly elastic at price equal to belief about average quality, $\mu = E(\theta)$. Recall θ is the utility to buyers from a car of that type.

4 Competitive equilibrium

- Rational beliefs and participation constraint lead us to the equilibrium.

- What is the belief μ about average quality?

- $\theta \sim U[2.0, 3.0]$, so let $f(\theta)$ be the pdf.

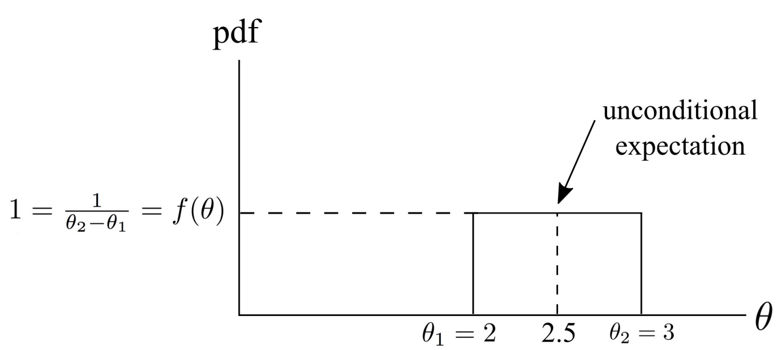

Figure 1: Distribution of car quality, and unconditional expectation.

- For the uniform distribution the height of the rectangle is: $f(\theta) = \frac{1}{\theta_2 - \theta_1}$ so that the area is one.

- In general,

$$\begin{aligned}
\int_{\theta_1}^{\theta_2} f(\theta)d\theta &= 1 \\
\int_{\theta_1}^{\theta_2} \frac{1}{\theta_2 - \theta_1} d\theta &= 1 \\
\frac{\theta_2}{\theta_2 - \theta_1} - \frac{\theta_1}{\theta_2 - \theta_1} &= 1
\end{aligned}$$

- For $\theta_1 = 2.0$, $\theta_2 = 3.0$, $f(\theta) = \frac{1}{3-2} = 1$, so buyer's unconditional expectation of θ is:

$$\begin{aligned}
E(\theta) &= \int_{\theta_1}^{\theta_2} f(\theta)\theta d\theta \qquad (3) \\
E(\theta) &= \int_2^3 \theta d\theta = \left.\frac{\theta^2}{2}\right|_2^3 \\
E(\theta) &= \frac{1}{2}(9-4) = 2.5
\end{aligned}$$

  - So, the buyers unconditional expectation of quality is $\mu = 2.5$.

- However, buyers are sophisticated in the following sense.

  - They recognize that sellers have reservation value $r(\theta) = \theta - 0.1$, and $r(\theta) \leq p$ implies $\theta \leq p + 0.1$.
  - Thus, if $p = \mu = 2.5$, then only cars on the interval $[2.0, 2.6]$ are for sale.
  - Thus, we have adverse selection — the high quality cars will not be available in the market.
  - This is what is known as a "shrinking pool" of potential cars for sale.

- Given this, buyers now have an expected quality of:

$$\begin{aligned}
E(\theta) &= \int_2^{2.6} \frac{1}{2.6-2} \theta d\theta \\
E(\theta) &= \left(\frac{5}{3}\right) \left.\frac{\theta^2}{2}\right|_2^{2.6} = 2.3
\end{aligned}$$

- But, given $p = \mu = 2.3$, only cars on the interval $[2.0, 2.4]$ are for sale.
  - The pool continues to shrink, losing the high quality end of the spectrum.
- Where does this process end?
  - We need to find the critical value where participation constraint and beliefs are mutually consistent.
  - Participation constraint: Up to what value of θ will participate in the market?
  - Let this critical value be x.
  - The uniform distribution on $[\theta_1, x]$ has density (height) : $\frac{1}{x-\theta_1}$.
  - The participation constraint is: $\mu = p \geq r(\theta)$, or:

$$\int_{\theta_1}^{x} \frac{1}{x-\theta_1} \theta d\theta \geq x - 0.1 \tag{4}$$

  - Here $\theta_1 = 2.0$, so

$$\int_{2}^{x} \frac{1}{x-2} \theta d\theta \geq x - 0.1$$

  - The critical value of x implies that this constraint holds with equality, so:

$$\begin{aligned}
\int_{2}^{x} \frac{1}{x-2} \theta d\theta &= x - 0.1 \\
\frac{1}{x-2} \left. \frac{\theta^2}{2} \right|_2^x &= x - 0.1 \\
\frac{x^2}{2} - 2 &= (x-2)(x-0.1) \\
\frac{x^2}{2} - 2 &= x^2 - 2.1x + 0.2 \\
\frac{x^2}{2} - 2.1x + 2.2 &= 0 \\
x &= 2, x = 2.2
\end{aligned}$$

  - Clearly, $x = 2.2$ is the solution since the pool is shrinking from above. Only cars on the interval $[2.0, 2.2]$ are for sale in equilibrium.

– The equilibrium is:

$$\begin{aligned} \mu &= p^* = \int_2^{2.2} \frac{1}{2.2-2}\theta d\theta = 2.1 \qquad (5) \\ \Theta^*(p^*) &= \{\theta : \theta \in [2.0, 2.2]\} \end{aligned}$$

– So, with asymmetric information, in equilibrium, only the worst $\frac{1}{5}$ of the cars are for sale.

– With perfect information every car would be sold.

– The competitive equilibrium is:

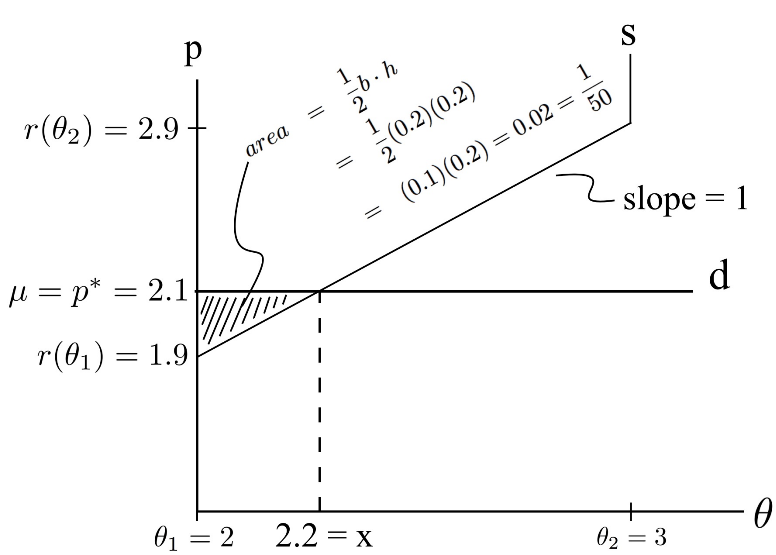

Figure 2: Lemons Equilibrium.

- The social surplus with perfect information is: $N(0.1) = \frac{N}{10}$, where N is the number of cars sold and \$100 was the social surplus per car.

  – With asymmetric information, the social surplus is: $\frac{1}{5}N(0.1) = \frac{N}{50}$.

- This example resulted in a partial (80%) market failure.

- Other possibilities are that no cars get sold (complete market failure), or there could be multiple equilibria with an obvious Pareto ranking.

- Definition of a competitive equilibrium with asymmetric information.
  - A competitive equilibrium when sellers hold asymmetric information is a (p^*, Θ^*) such that:

1. $\Theta^* = \{\theta : r(\theta) \leq p^*\}$ are the participants and
2. $p^* = E(\theta|\theta \in \Theta^*) = E(\theta|p^*)$, so p^* clears the market.

- An example much in the news these days is health insurance. If the healthiest members withdraw from the plan, the pool shrinks, the expected cost per member increases, price increases, and the pool shrinks further. Again, we have a partial or total market failure.

5 Behavioral Considerations

People often have a hard time finding optimal actions (or BNE strategies) in the presence of adverse selection. For example, in the takeover game proposed in the Prologue, the Target's refusal of bids b below its true value v means that the Acquirer gets only the less valuable targets.

Given that BR by the Target, the payoff for the Acquirer who bids b is $1.5v - b$ when $v < b$ and is 0 when $v \geq b$. With v uniformly distributed on $[0, 100]$, the expected payoff is

$$\pi(b) = \int_0^b [1.5v - b]dv = .75b^2 - b^2 = -.25b^2 \tag{6}$$

which is maximized at $b = 0$.

People who play this game repeatedly typically start out bidding too high; a typical median bid across A players in early periods is around 50 or 60. Over time they learn to decrease their bids, but the median usually stalls out around 20 or 30 and never gets close to 0, even after dozens or even hundreds of periods!

Why? Selten (2004) suggests that the reason is "directional learning." After a trial when a player loses money (i.e., when $1.5v < b$), he tends to bid lower next period. But after a period when he would have made money if he had bid higher (i.e., when $v > b$) he tends to increase his bid. These "impulses" to increase or decrease bids seem to balance at around $b = 20$ or 30 for typical subjects.

See the literature on "cursed equilibria" for further discussion. The "winner's curse" in common value auctions is the leading example.

6 Further Reading

Akerlof, G. (1970). "The Market for Lemons: Quality Uncertainty and the Market Mechanism," *The Quarterly Journal of Economics*, 84(3): 488-500.

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Rothschild, M. and Stiglitz, J. (1976). "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *The Quarterly Journal of Economics*, 90(4): 629-49.

Selten, Reinhard (2004). "Learning direction theory and impulse balance equilibrium." Chapter 12 in D. Friedman and A. Cassar, Economics Lab: An Intensive Course in Experimental Economics, Routledge.