Fisherman Theory: Notes

Refs:
- FM, Ch 17 (Fallegri & Molynex)

These notes will begin with the two-date barter model, and later extend it to several dates and monetary exchange.

The Basic Model

2 dates: 0 (now), 1 (later). No uncertainty.

1 good: C ("corn" or "consumption basket")

N agents, i = 1, ..., N, each with given preferences, represented by \( U(c_0, c_1) \), with classical properties (smooth, monotone, s.g.-concave).

\[
\begin{align*}
\text{MRS}_i &= \frac{\partial U}{\partial c_0} \\
\text{MU}_0 &= \frac{\partial U}{\partial c_0} \\
\text{MU}_1 &= \frac{\partial U}{\partial c_1}
\end{align*}
\]

- slope = \( \text{MRS}_i = 1 + \text{MRTP}_i \)

\[
\frac{\text{MRTP}_i}{\text{MU}_i} = \frac{\text{MU}_0}{\text{MU}_1} - 1
\]

The marginal rate of time preference (MRTP) summarizes the trade-off for i between present and future consumption:

on margin, i demands MRTP_i units extra future consumption per unit of foregone current consumption.

and
productive opportunities, represented by a production function

\[ y = f(x) \]

relating increments to future consumption \( y = \Delta c_t \) to foregone current consumption \( x = -\Delta c_0 \).

Increments are taken relative to an endowment point \( E = (e_0, e_1) \).

so that \( f(0) = 0 \) and \( f' > 0 \). Also, assume \( f'' < 0 \).

\[ \text{MRT} = 1 + \text{MROI} = f'(x). \]

Also, assume a frictionless financial market in which \( c_0 \) can be exchanged for claims on \( c_1 \) at date 0. Each unit of \( c_0 \) exchanges for \((1+r)\) units of \( c_1 \), so \( r \) is the (real) interest rate.

The exchange is called

\( C_0 \xrightarrow{(lending)} c_0 \xrightarrow{1+r} C_1 \)

\( C_1 \xrightarrow{(borrowing)} C_0 \xrightarrow{\text{lending}} (c_0) \)

\( \frac{C_1}{1+r} \)
Analysis of Model.

(i) Wealth and present value. For $C = (c_0, c_1)$ and $r$ given, define $PV(C)$ to be the $c_0$-intercept of the budget line through $C$. It is the wealth, or present value, of the bundle $C$. Since 1 unit of $c_0$ exchanges for $(1+r)$ units of $c_1$, the agent can get $c_1 / (1+r)$ units of $c_0$. Hence

$$w = PV(C) = c_0 + \frac{c_1}{1+r}.$$ 

II. Individual optimizes given $r$ and own characteristics $(E$ and $f_i)$.

(ii) Optimal investment. Given productive opportunities, preferences $(U_i)$, and the market interest rate $r$, the agent wishes to maximize wealth, because this provides the greatest consumption opportunities.

Formally:

$$\max_x \quad w = PV(Q) = q_0 + \frac{q_1}{1+r} = e_0 - x + \frac{e_1 + f(x)}{1+r}.$$ 

The FOC is $0 = \frac{dw}{dx} = -1 + \frac{f'(x)}{1+r} \implies 1+r = f'(x) = 1+MROI \implies [r = MROI] \text{ at optimal investment level } x \text{ (interior solution)}$.

Rule: Invest to max $PV$:

$m$, $Q$ : MROI = \(r\),

$PV$ increases $\max_{i}$ when $MROI > r$ (decreases) $(<)$

NB: Fisher Separation Theorem:

Optimal $Q$ is independent of preference $U_i$:

Everyone expects MROI to $r$ = social efficiency joint ownership.
(2) Optimal Consumption / Borrowing / Lending. The agent, having moved her budget line out as far as possible by choosing \( Q \) optimally, now wishes to find most preferred point \( C \) on budget line. Since a bundle can be exchanged in the financial market for any other bundle with the same present value, we have:

\[
\text{Max} \, U(c_o, q_i) \, \text{s.t.} \, \text{PV}(C) = \text{PV}(Q) = y, \quad \text{where} \quad C = Q + B,
\]

where \( B = (b, -(1+r)b) \) is the borrowing - repayment vector. 

This can be rewritten as

\[
\text{Max} \, U(b_0 + b, q_i - (1+r)b). \quad \text{The FOC is}
\]

\[
O = (MU_0)(1) + (MU_1)(-1) \Rightarrow 1 + r = \frac{MU_0}{MU_1} = \text{MRS} = 1 + \text{MRTP}
\]

\[
\Rightarrow r = \text{MRTP} \, \text{at optimal interior consumption C (or borrowing b)}.
\]

\[\text{Rule: Increase borrowing when } r < \text{MRTP, decrease.}\]

1. **NB:** At optimum, every agent \( i \) has MRTP equal to \( r \), after sufficient borrowing or lending. (TECHNICALITY: impose Imada cond + classical approach to avoid corner solution - very unrealistic here.)

2. **Fisher Separation Theorem:** choice of \( Q \) is market - independent of prebs. \( \Rightarrow \) joint overprice.
(3) Equilibrium real (riskless) interest rate.

Each agent $i$ equates MRTP and MROI to $r$, which involves some borrowing ($b_i > 0$) or lending $b_i = -l_i > 0$. An increase in $r$ will decrease $b_i$ (or increase $l_i$) via the production effect ($f'' < 0$) and the usual substitution effect ($U$ concave). If $b_i, l_i, c_0$ and $c_1$ are both normal goods, the income effect will also decrease $b_i$ if $b_i > 0$ but will decrease $l_i$ if $l_i > 0$. The net effect is presumably as drawn (with $l_i$ possibly backward-bending at very high $r$).

Aggregate borrowing is $B(r) = \sum_{i:b_i > 0} b_i(r)$ and aggregate lending is $L(r) = \sum_{i:l_i > 0} l_i(r)$. Under present presumptions, there is a unique $r^*$ such that $B(r^*) = L(r^*)$. This is the equilibrium real interest rate, or (as we will later call it) the riskless rate.

(4) Comparative statics: $r^*$ is increased by increased productive opportunities (MROI ↑) and by increased impatience (MRTP ↑).

See exercises, use autarky model ←