Fisherian Theory: Notes

Refs: • Hirshleifer[2], Price Theory and Applications, ~~4th Ed~~ or ~~5th~~ 6th ed: Ch14.

• Copeland & Weston, Financial Theory and Corporate Policy, 3rd Edition, Ch1-2.

• FM Ch 17 (Fabozzi & Modigliani)

These notes will begin with the two-date barter model, and later extend it to several dates and monetary exchange.

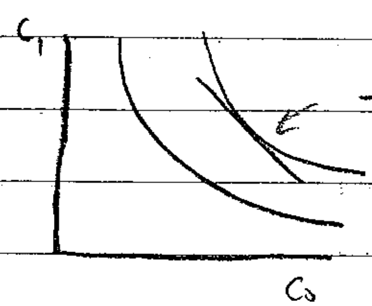
The Basic Model

2 dates: 0 (now), 1 (later). No uncertainty.

1 good: c ("corn" or "consumption basket")

N agents, $i = 1, \ldots, N$, each with given:

preferences, represented by $\mathcal{U}(c_0, c_1)$, with classical properties (smooth, monotone, s. q.-concave, Inada)



$$\boxed{\begin{aligned} MU_0 &= \frac{\partial \mathcal{U}}{\partial c_0} \\ MU_1 &= \frac{\partial \mathcal{U}}{\partial c_1} \end{aligned}}$$

$-\text{slope} = MRS_i = 1 + MRTP_i$

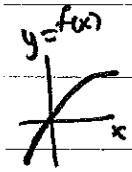
$$MRTP_i = \frac{MU_0}{MU_1} - 1$$

The Marginal rate of time preference (MRTP) summarizes the trade-off for i between present and future consumption: on margin, i demands $MRTP_i$ units extra future consumption per unit of foregone current consumption;

and

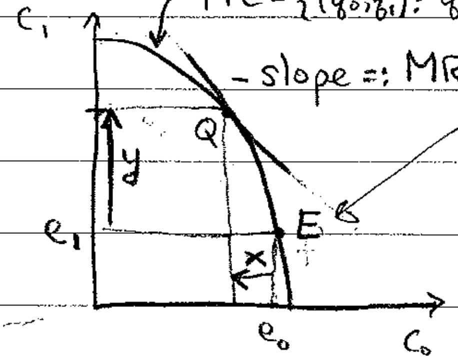
and

productive opportunities, represented by a production function $y = f(x)$ relating increments to future consumption $y = \Delta c_1$ to foregone current consumption $x = -\Delta c_0$.



Increments are taken relative to a given endowment point $E = (e_0, e_1)$ so that $f(0)=0$ and $f' > 0$. Also, assume $f'' < 0$.

$PPC = \{(q_0, q_1): q_0 = e_0 - x,\ q_1 = e_1 + f(x)\} \cap R^2_+$



$-$slope $=: MRT =: 1 + MROI = f'(x)$.

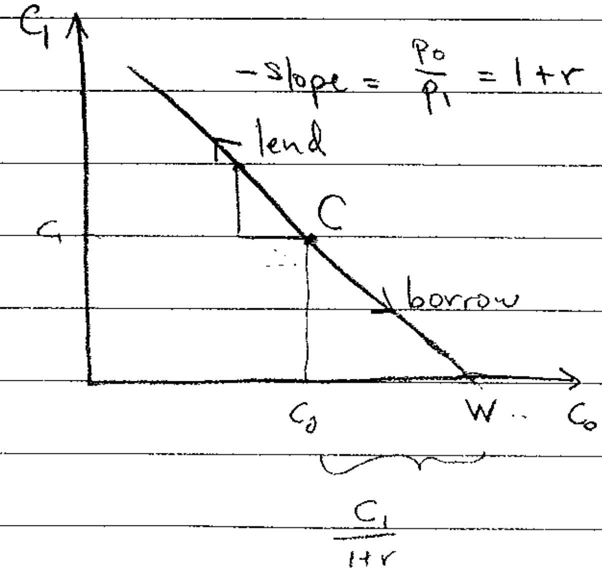
$y \ \frac{df}{dx} \nearrow$

$ROI := f(x) - x$, $AROI = \frac{f(x)}{x} - 1$, and so

$MROI = f'(x) - 1$ is marginal return on investment

Also, assume a frictionless financial market in which c_0 can be exchanged for claims (promises) on c_1, at date 0. Each unit of c_0 exchanges for $(1+r)$ units of c_1, so $r =$ the (real) interest rate.

secure - no uncertainty.



$c_0 \xrightarrow{1+r} c_1$

The exchange is called borrowing (lending) when $\Delta c_0 > 0$ ($<$).

Analysis of Model.

(0) Wealth and present value. For $C = (c_0, c_1)$ and r given, define $PV(C)$ to be the c_0-intercept of the budget line thru C. It is the wealth, or present value of the bundle C. Since 1 unit of c_0 exchanges for $(1+r)$ units of c_1, the agent can ~~got~~ exchange c_1 for $c_1/(1+r)$ ~~more~~ additional units of c_0. Hence

$$w = PV(C) = c_0 + \frac{c_1}{1+r}.$$

II. Individual optimum, given r + own characteristics

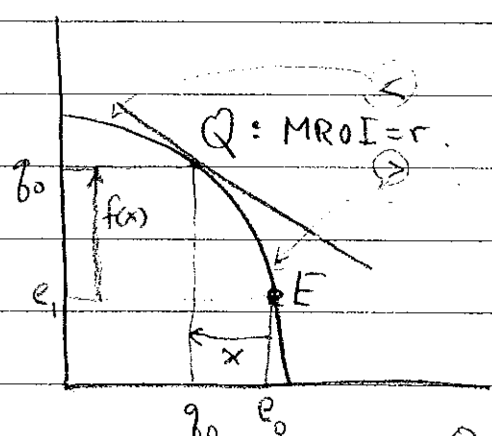
(1) Optimal investment. Given productive opportunities (E and f_i), preferences (U_i) and the market interest rate r, the agent wishes to maximize wealth, because this provides the greatest consumption opportunities. Formally:

$$\max_x \; w = PV(Q) = q_0 + \frac{q_1}{1+r} = e_0 - x + \frac{e_1 + f_i(x)}{1+r}.$$

The FOC is $0 = \frac{dw}{dx} = -1 + \frac{f_i'(x)}{1+r} \Rightarrow 1 + r = f'(x) = 1 + MROI$

$\Rightarrow \boxed{r = MROI}$ at optimal investment level x (interior solution)

q



Rule: Invest to max PV:

PV increases (decreases) in x when MROI > r (<).

NB: Fisher Seperation Theorem:

Optimal Q is independent of preference U_i.

Everyone equates MROI to r ⇒ social efficiency.

joint ownership.

(2) Optimal Consumption / Borrowing / Lending. The agent, having moved her budget line out as far as possible by choosing Q optimally, now wishes to find most preferred point C on budget line. Since a bundle can be exchanged in the financial market for any other bundle with the same present value, we have:

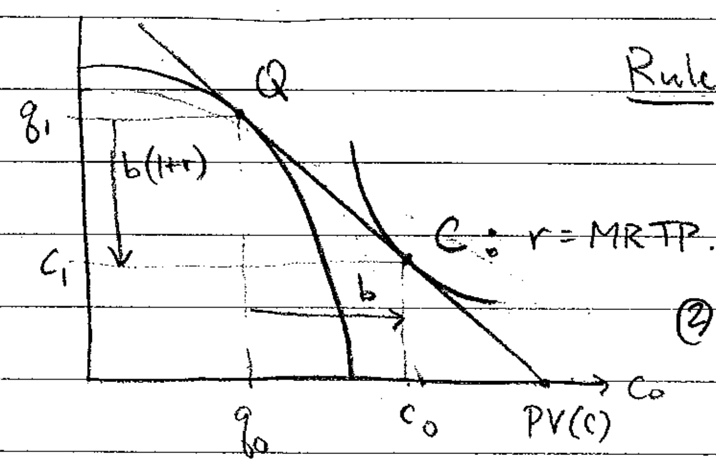
given U, Q and r,

$\max_{c_0, c_1 \geq 0} U(c_0, c_1)$ s.t. $PV_r(C) = PV_r(Q) = w$. ~~Since consumption $C = Q + B$, where $B = (b, -(1+r)b)$ is the borrowing-repayment vector,~~ this can be rewritten as

$$\max_b U_i(q_0 + b,\ q_1 - (1+r)b).$$ The FOC (nec. & suff. here :: strict quasi convexity + INada) is

$$0 = (MU_0)(1) + (MU_1)(-(1+r)) \Rightarrow 1 + r = \frac{MU_0}{MU_1} = MRS = 1 + MRTP$$

$\Rightarrow$ $\underline{\underline{r = MRTP}}$ at optimal interior consumption C (or borrowing b).



Rule: Increase borrowing when $r < MRTP$
(decrease) (>)

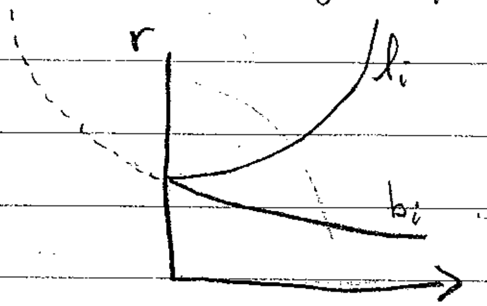
(2) $\longrightarrow b(r)$
- prod effect & sub effect $\Rightarrow$ (−)
- income effect for normal c_0, c_1 switches sign @ $b = 0$.

(1) NB: At optimum, every agent i has MRTP equal to r, after sufficient borrowing or lending. (technicality: impose Inada conds + classical (non-smooth) convex to avoid corner solns - very reasonable here).

(3) Fisher Separation Thm:
- choice of Q is unanimous - indep of prefs. $\Rightarrow$ joint ownership works
- choice of C is individual

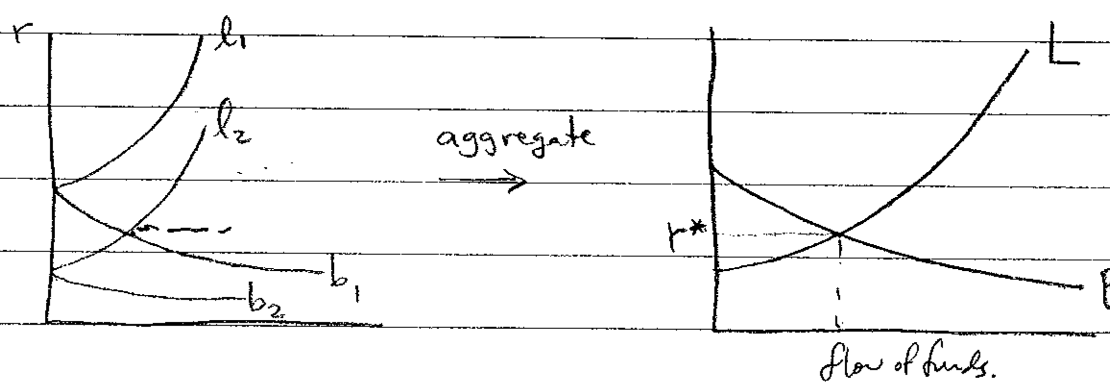
(3) Equilibrium real (riskless) interest rate.

Each agent i equates MRTP and MROI to r, which involves some borrowing ($b_i>0$) or lending $l_i=-b_i\geq 0$. An increase in r



will decrease b (or increase l) via the production effect ($f''<0$) and the usual substitution effect (u concave). If c_0 and c_1 are both normal goods, the income effect will also decrease b_i if $b_i>0$ but will decrease l_i if $l_i>0$. The net effect is presumably as drawn (with l_i possibly backward-bending at very high r).

Aggregate borrowing is $B(r)=\sum_{i:b\geq 0}^{N} b_i(r)$ and aggregate lending is $L(r)=\sum_{i:l>0} l_i(r)$. Under present presumptions, there is a unique r^* such that $B(r^*)=L(r^*)$. This is the equilibrium real interest rate, or (as we will later call it, the riskless rate.



NB. Not like S&D. Both L & B shift when underlying determinants shift (PPC, Prefs)

(4) Comparative statics: r^* is increased by increased productive opportunities (MROI ↑) and by increased impatience (MRTP ↑). See exercises. → use autarky model ←