UCSC Evolutionary Game Theory Class

Winter 2017.

Midterm Exam

Instructions. Write your answer to each question on a separate piece of paper. Write your name at the top of each page. Points as marked; total is 50. Generous partial credit for **brief** answers that display relevant knowledge but are incomplete.

1. Consider the 3x3 single population evolutionary game defined by the payoff matrix

$$w = \begin{pmatrix} 0 & 3 & -1 \\ 3 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$$

   a. Write out the three Delta functions $\Delta w_{1-2}(s), \Delta w_{1-3}(s), \Delta w_{2-3}(s)$, where $s = (s_1, s_1, s_1)$ is the state (or shares) vector. [4 pts]

   b. Use those functions to sector the simplex, drawn as an equilateral triangle. [4 pts]

   c. To each sector of the simplex, assign a range (cone) of directions for the trajectory tangent vector $\dot{s}$ under monotone dynamics. [4 pts]

   d. Write down a 2x2 submatrix for each of the three edge games, identify the type of each (e.g., DS or CO), and mark the trajectories on the corresponding edge of the simplex in part b. [4 pts]

   e. Find all steady states of monotone dynamics for w and (to the extent possible using the results of previous parts of this problem) identify each as a source, sink, center or saddle. [4 pts]

   f. Find all Nash equilibria (NE) and ESS (evolutionarily stable strategies) of w. For full credit, verify the relevant inequalities.[4 pts]

   g. Write out the system of three ordinary differential equations for replicator dynamics of w. [3 pts]

   h. Explain how you would find all stable steady states under replicator dynamics for the present example if you had plenty of time.[3 pts]

   i. For extra credit, if time does permit, find all stable steady states and their basins of attraction. [3 pts max]

2. This problem asks you distinguish between single and multiple population games.

   a. Population 1 has two pure strategies. Write down a matrix specifying strategic interaction within that population, assuming no interactions with other populations. [2 pts]

   b. Population 2 has three pure strategies. Write down a matrix specifying strategic interaction within that population, assuming no interactions with other populations. [2 pts]

   c. Now assume that, contrary to parts (a. , b.) above, the payoff to each strategy in each population depends on the other population state, but not its own. Write down a matrix (or bi- or multi-matrix) that might capture this situation. [2 pts]

   d. Now suppose that all effects in previous parts of this question may be important. What sort of matrices (or multi-matrices) can capture the strategic interaction? [2 pts]

   e. Suppose that there is a third population that has two pure strategies. If there are no own-population effects, what sort of matrices (or multi-matrices) can capture the strategic interaction? [2 pts]

3. This problem is about assortative matching.

   a. What is the implicit assumption in the previous problems about how strongly players using a given strategy interact with players who use different strategies? [4 pts]

   b. The text mentions three different ways to relax that implicit assumption. Briefly discuss the advantages of each. Which (if any) seems most relevant to your group's project? [6 pts]