# Econ 204B: Midterm Answer Key

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1. Consider the two player game described by the following payoff bimatrix.

| | a | b | c |
|---|---|---|---|
| $A$ | $\underline{3},\underline{4}$ | 4,2 | $\underline{8}$,2 |
| $B$ | 1,5 | $\underline{5},\underline{6}$ | 3,1 |
| $C$ | 2,1 | 3,$\underline{3}$ | 6,2 |

(a) Does either player have a dominant strategy?

(b) Does either player have a dominated strategy?

(c) Which strategy profiles survive iterated deletion of strictly dominated strategies? Be explicit on which strategies dominate the deleted strategies.

(d) Find all Nash equilibrium(NE) in pure strategies, if any.

(e) Find all Nash equilibrium(NE) in mixed strategies, if any.

**Answer:**

(a) No. After underlining the best response to each player, there is no dominant strategy.

(b) Yes. For player 1, $C$ is strictly dominated by $A$.
(Also, $c$ is dominated by some mixes of $a$ and $b$.)

(c) Since $C$ is strictly dominated by $A$ for player 1, delete $A$ first. After removing $A$, $c$ is strictly dominated by $a$ for player 2, then delete $c$. The remaining payoff bimatrix:

| | a | b |
|---|---|---|
| $A$ | 3,4 | 4,2 |
| $B$ | 1,5 | 5,6 |

(d) Underline best response to each player. There are two pure strategy NE: $\{(A, a), (B, b)\}$.

| | a | b |
|---|---|---|
| $A$ | 3,4 | 4,2 |
| $B$ | 1,5 | 5,6 |

(e) Assume player 1's mixed strategy is $\sigma_1 = pA + (1-p)B$ where $p$ is the probability of choosing $A$.
Also, player 2's mixed strategy is $\sigma_2 = qa + (1-q)b$ where $q$ is the probability of choosing $a$.
For player 1:
$$f_1(A, \sigma_2) = f_1(B, \sigma_2) \quad \Rightarrow \quad 3q + 4(1-q) = q + 5(1-q) \quad \Rightarrow \quad q* = \frac{1}{3}$$
For player 2:
$$f_2(a, \sigma_1) = f_2(b, \sigma_1) \quad \Rightarrow \quad 4p + 5(1-p) = 2p + 6(1-p) \quad \Rightarrow \quad p* = \frac{1}{3}$$
The mixed NE is $(\frac{1}{3}A + \frac{2}{3}B, \frac{1}{3}a + \frac{2}{3}b)$.

2. Consider the Bernoulli function $u(x) = \sqrt{(10+x)}$. Compute the coefficient of absolute risk aversion and the coefficient of relative risk aversion.

**Answer:**
$u'(x) = \frac{1}{2}(10+x)^{-\frac{1}{2}}$
$u''(x) = -\frac{1}{4}(10+x)^{-\frac{3}{2}}$
The coefficient of absolute risk aversion is $A(x) = -\frac{u'(x)}{u''(x)} = \frac{1}{2(10+x)}$.
The coefficient of relative risk aversion is $R(x) = xA(x) = \frac{x}{2(10+x)}$.

3. Consider the following lotteries written in ([prize], [probability]) vector notation: $A = ([0,5],[0,1]), B = ([0,5],[.5,.5]), C = ([0,10],[.5,.5])$, and $D = ([0,5,10],[.2,.6,.2])$. E.g. A pays 5 for sure while D pays either 0, 5 or 10 with respective probabilities 20%, 60% and 20%. To the extent possible, rank the lotteries by

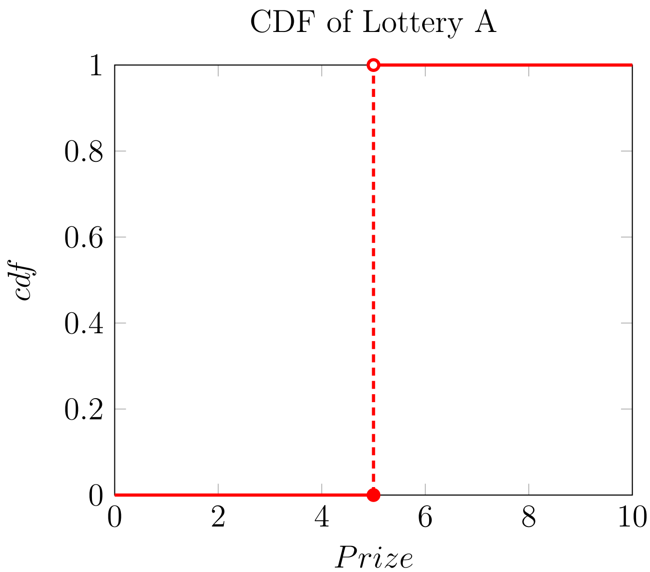
(a) first order stochastic dominance

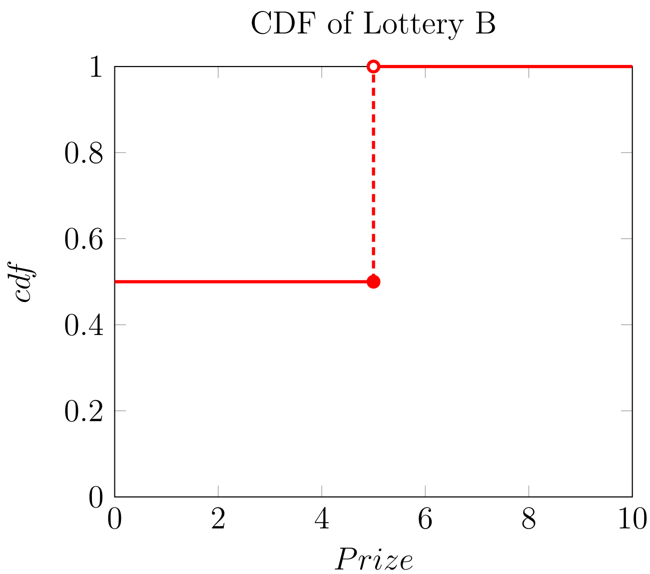
(b) second order stochastic dominance

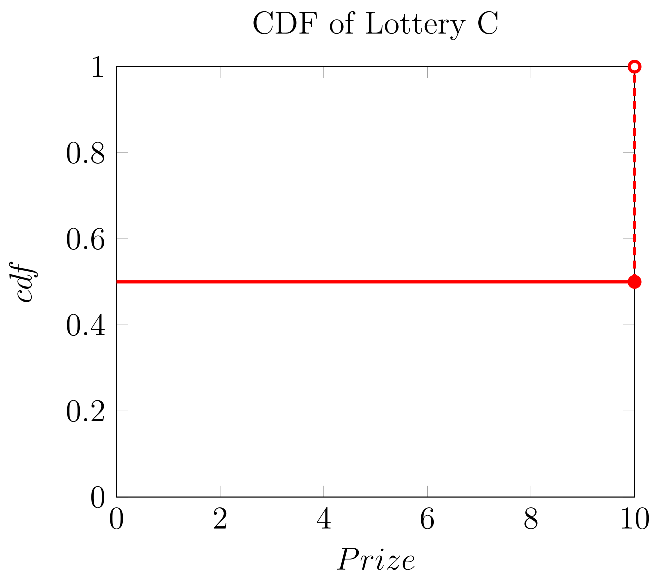
(c) unanimous preference by all expected utility maximizers

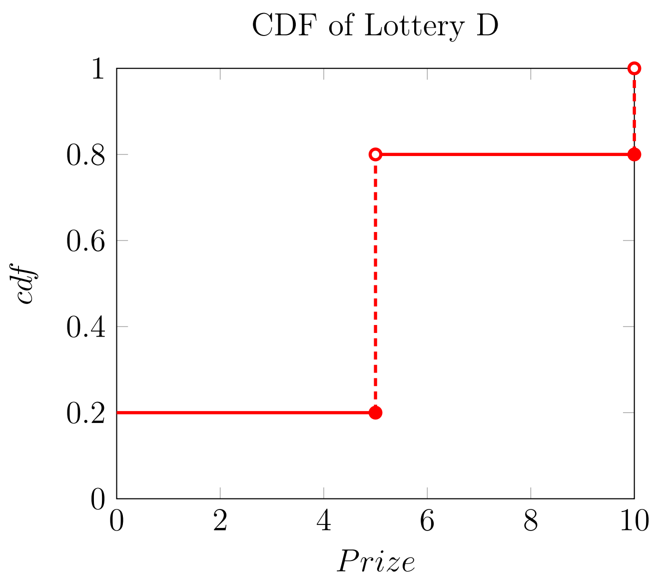
(d) unanimous preference by all risk-averse expected utility maximizers

**Answer:**


CDF of Lottery A
1
0.8
0.6
0.4
0.2
0
cdf
0
2
4
6
8
10
Prize


CDF of Lottery B
1
0.8
0.6
0.4
0.2
0
cdf
0
2
4
6
8
10
Prize




(a) From the CDF's of the lotteries, we can see that CDF of the lottery B is never below the CDF's of any of other lotteries.
$A \succ_{FOSD} B$
$C \succ_{FOSD} B$
$D \succ_{FOSD} B$
This is the furthest we can go with the first order stochastic dominance and we cannot compare the first order stochastic dominance between A, C and D.

(b) Since the expected payoff of A, C and D are the same ($E[A] = E[C] = E[D] = 5$), we can compare the second order stochastic dominance between these. We can see that the area below the CDF of C is always greater than or equal to that of D

and A. The area below the CDF of D is always of greater than or equal to that of A.
$A \succ_{SOSD} C$
$A \succ_{SOSD} D$
$D \succ_{SOSD} C$
Therefore, $A \succ_{SOSD} D \succ_{SOSD} C$

(c) From the result from (a), we can tell that any expected utility maximizer would prefer lotteries A, C or D over B since B is a first order stochastic dominated lottery but we cannot tell the rank between A, C and D. This depends on the risk attitude of the decision maker.
$A \succ B$
$C \succ B$
$D \succ B$

(d) By the result from (b), any risk-averse expected utility maximizer would prefer the lotteries in the rank of second order stochastic dominance and we know that from (c) that any expected utility maximizer would not prefer the first order stochastic dominated lottery.
$A \succ D \succ C \succ B$

4. The n-player Ninety percent Guessing Game (NGG) pays 5 to the player whose guess is closes to 90% of the average guess. Formally, players simultaneously choose $x_i$ in $S_i = [0, 1]$. Then $u_i = 0$ unless $i = argmin_j \mid x_j - 0.9m \mid$, where $m = \Sigma \frac{x_j}{n}$, and $u_i = 5$ in that case. For now, suppose that there are $n = 10$ players

   (a) Which strategies in [0,1], if any, are strictly dominated?

   (b) In the reduced games obtained by deleting those dominated strategies, which other strategies, if any, any are strictly dominated?

   (c) Does the NGG game have an IDDS solution? If so, write it out. In any case, find all NE (Nash equilibria).

   (d) Now suppose that $n = 2$. How do your answers to parts a-c change?

   **Answer:**

   (a) Guessing any number that lies above 0.9 is dominated by 0.9 for every player since it cannot possibly be 90% of the average of any guess. These can be eliminated.

   (b) In the reduced game obtained from (a), new possible strategy set would be $S_i' = [0, .9]$. Then from the new action set, any number that lies above 0.81 is dominated by 0.81 since no player will guess above 0.9 and 90% of 0.9 is 0.81. Again, these can be eliminated.

(c) We can delete the numbers that lie above 0.729 from $S_i'' = [0, 0.81]$. We continue this process — deleting all numbers above $.9^k$ ub stage $k$ — until all numbers above $\lim_{k\to\infty} = 0$ have been eliminated. Thus 0 is the IDDS strategy for every player. Every player guessing 0 is the unique Nash equilibrium from IDDS solution.

(d) If the number of the players are reduced down to two, we see that 0 is a (weakly) dominant strategy since the best response to the other player's guess is to guess any number below that number: $BR_i(s_{-i}) = [0, s_{-i})$. The only mutual best response is to choose 0, i.e., the unique Nash equilibrium for this case is $(s_1, s_2) = (0, 0)$.

5. The professor of a Monday-Wednesday class announces that she will give a quiz someday next week, but the particular day(M or W) will be a surprise. A student argues that surprise is impossible: if the quiz is on W, it will not be a surprise since no other options remain. So it can't be W. But now M won't be a surprise either, since W has been ruled out. The student concludes that there will be no exam and doesn't study.[Here's what actually happened. The professor gave the quiz on Monday and the student was unpleasantly surprised!] For many years, philosophers and logicians have puzzled over this apparent paradox,

   (a) Write out a two player extensive form game(EFG) in which player 1, the professor(P), secretly chooses the day in advance, and player 2, the student(S), guesses each day before class whether or not the exam is today(T) or later(L). Say the payoff is +1 to S and -1 to P each time the student guesses correctly, and is the opposite each time S guesses incorrectly. For example, if the exam is on W and the student guessed T on both M and W, then P's payoff is $1 - 1 = 0$ and S's payoff is $-1 + 1 = 0$.

   (b) Write out the sets of pure strategies for both players, and the normal form game(NFG) bimatrix.

   (c) Find all pure strategy Nash Equilibrium of the game (or show that none exist), using the bimatrix.

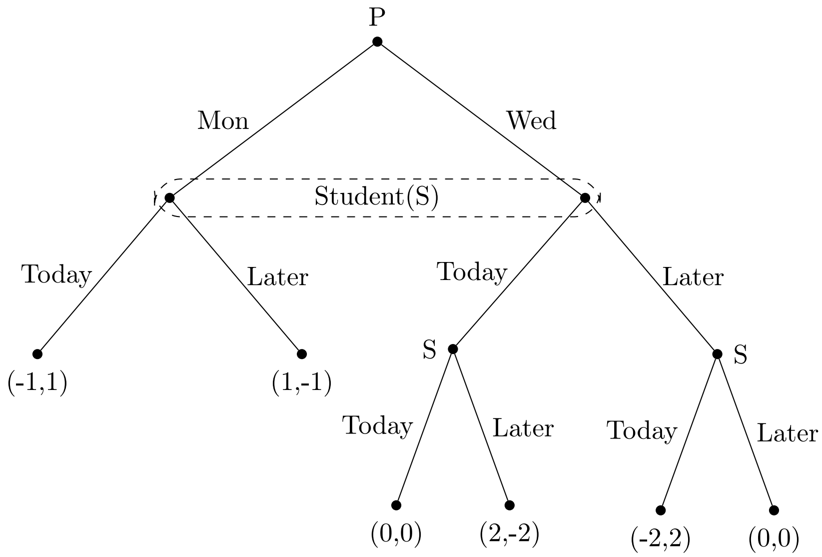
   (d) Find all mixed strategy Nash Equilibrium of the game, or show that none exist.

   (e) Find all SPNE of the EFG in part a.

   (f) For extra credit, write out the EFG for the 3 date version (MWF class). If you have time to kill, find all NE for this game.

   **Answer:**

   (a) EFG for the game can drawn as follows, with the lowest S nodes referring to Wednesday and the higher S nodes referring to Monday.




(b) Let $S_p$ and $S_s$ be the strategy sets for Player 1 and 2. $S_p$= {Monday, Wednesday} $S_s$= {TTT, TTL, TLT, TLL, LTT, LTL, LLT, LLL}, where the first item in the string refers to the Monday info set move, the second refers to the move at player 2's lower left node, and the last item in the string refers to her move at her lower right node.

The NFG can be drawn as follows:

| | **TTT** | **TTL** | **TLT** | **TLL** | **LTT** | **LTL** | **LLT** | **LLL** |
|---|---|---|---|---|---|---|---|---|
| **Monday** | (-1,1) | (-1, 1) | (-1, 1) | (-1, 1) | (1,-1) | (1,-1) | (1,-1) | (1,-1) |
| **Wednesday** | (0,0) | (0,0) | (2, -2) | (2, -2) | (-2,2) | (0,0) | (-2, 2) | (0,0) |

(c) Underlining the best responses for each player in response to other player's strategies. Since no strategy is a mutual best response, there is no pure strategy Nash Equilibrium.

| | **TTT** | **TTL** | **TLT** | **TLL** | **LTT** | **LTL** | **LLT** | **LLL** |
|---|---|---|---|---|---|---|---|---|
| **Monday** | (-1,$\underline{1}$) | (-1, $\underline{1}$) | (-1,$\underline{1}$) | (-1, $\underline{1}$) | ($\underline{1}$,-1) | ($\underline{1}$,-1) | ($\underline{1}$,-1) | ($\underline{1}$,-1) |
| **Wednesday** | ($\underline{0}$,0) | ($\underline{0}$,0) | ($\underline{2}$,-2) | ($\underline{2}$,-2) | (-2, $\underline{2}$) | (0,0) | (-2, $\underline{2}$) | (0,0) |

(d) The above NFG can be reduced by deleting columns that are repeated. We get the following reduced form NFG -

| | **TT*** | **TL*** | **L*T** | **L*L** |
|---|---|---|---|---|
| **Monday** | (-1,$\underline{1}$) | (-1, $\underline{1}$) | ($\underline{1}$,-1) | ($\underline{1}$,-1) |
| **Wednesday** | ($\underline{0}$,0) | ($\underline{2}$,-2) | (-2, $\underline{2}$) | (0,0) |

Column L*L can be deleted since it is dominated for player 2 (student) by various mixes of **L*T** and **TT***. So we are left with -

| | **TT*** | **TL*** | **L*T** |
|---|---|---|---|
| **Monday** | (-1,$\underline{1}$) | (-1, $\underline{1}$) | ($\underline{1}$,-1) |
| **Wednesday** | ($\underline{0}$,0) | ($\underline{2}$,-2) | (-2, $\underline{2}$) |

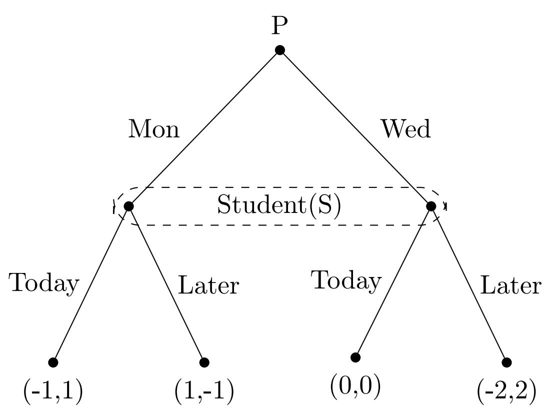
(We soon see that **TL*** can't be part of a SPNE, so we should focus on mixes that give it zero weight.) Assume that Professor randomizes between strategies Mon and Wed using $p$ and $(1-p)$ and the student randomizes between {TT*, TL*, L*T} using $q_1$, $q_2$ and $(1-q_1-q_2)$ respectively.

$$BR_1(q) = \begin{cases} \text{Play \{Wed\} if } 4q_1 + 6q_2 > 3 \\ \text{Play \{Mon\} if } 4q_1 + 6q_2 < 3 \\ \text{Play \{Mon, Wed\} with } 0 \leq p \leq 1 \text{ if } 4q_1 + 6q_2 = 3 \end{cases}$$

$$BR_2(p) = \begin{cases} \text{Play \{L*T\} if } 0 \leq p < \frac{1}{2} \\ \text{Play \{TT*, L*T\} if } p = \frac{1}{2} \\ \text{Play \{TT*\} if } \frac{1}{2} < p < 1 \\ \text{Play \{TT*, TL*\} if } p = 1 \end{cases}$$

The mutual best responses include $q_1 = \frac{3}{4}$, $q_2 = 0$ and p $= \frac{1}{2}$. Therefore, the mixed strategy Nash Equilibrium is given by $\{\frac{1}{2}\text{M} + \frac{1}{2}\text{W}, \frac{3}{4}\text{TT*} + \frac{1}{4}\text{L*T}\}$

(e) There are two proper subgames in this game. Solving them using BI we get the following tree -




To solve for SPNE, we will covert the above tree into a normal form game and solve for nash equilibrium. We get the following NFG -

| | **T** | **L** |
|---|---|---|
| **Monday** | (-1,$\underline{1}$) | ($\underline{1}$,-1) |
| **Wednesday** | ($\underline{0}$,0) | (-2, $\underline{2}$) |

By looking at the Best Responses (underlined in the table above) for both the players, we see that there are no pure strategy nash equilibrium. Solving for mixed nash equilibrium.

Assume that Professor randomizes between strategies Mon and Wed using $p$ and $(1-p)$ and the student randomizes between $\{$T, L$\}$ using $q$ and $(1-q)$ respectively.

$E_1(M, q)$ = (-1)*q + 1*(1-q) = -q +1 -q = 1-2q

$E_1(W, q)$ = (0)*q + -2*(1-q) = -2+2q

$E_1(M, q) = E_1(W, q) \Rightarrow$ 1-2q = -2 + 2q

$\Rightarrow$ q = $\frac{3}{4}$

$E_2(T, p)$ = (1)*p + 0*(1-p) = p

$E_1(L, p)$ = (-1)*p + 2*(1-p) = -p + 2 - 2p = 2-3p

$E_1(T, p) = E_1(L, p) \Rightarrow$ p = 2-3p

$\Rightarrow$ p = $\frac{1}{2}$

Therefore, SPNE is playing mixed Strategy profile (p,q) = ($\frac{1}{2}$, $\frac{3}{4}$)