Econ 204B Midterm Answer-key

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1. Consider the lottery L that pays 10 with probability 0.6 and pays 1 otherwise.

a. What is the mean and variance of this lottery?

$$\mu(L) = 10 \times 0.6 + 1 \times 0.4 = 6.4$$

$$Var(L) = 0.6 \times (10 - 6.4)^2 + 0.4 \times (1 - 6.4)^2 = 19.44.$$

b. Write down a lottery that first-order stochastically dominates L.

Lottery M= (10,0.8; 1, 0.2) FOSD lottery L.

c. What is the most that a person with Bernoulli function u(x)=log x would pay to play lottery L? What is her risk premium?

$E_L(u) = 0.6 \times log(10) + 0.4 \times log(1) = 0.6.$

$u(CE) = E_L(u) \implies CE = 10^{0.6} \approx 3.98.$

Risk premium$\pi = \mu(L) - CE \approx 6.4 - 3.98 = 2.42.$

d. Compute this person's coefficients of absolute and of relative risk aversion at x=the mean of L. Which coefficients (if either) has the same value at x=100?

$$ARA = -\frac{u"(x)}{u'(x)} = -\frac{-1/x^2}{1/x} = \frac{1}{x}$$

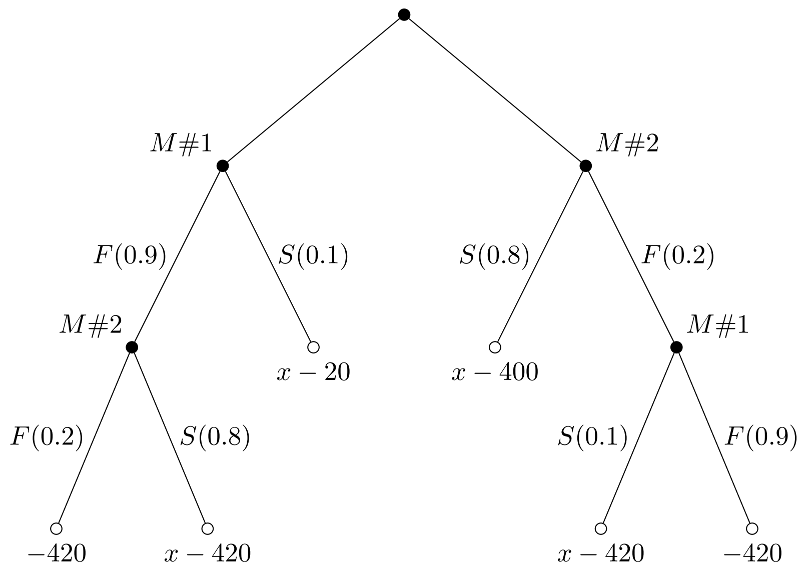
at $x = \mu(L)$, $ARA = \frac{1}{6.4} = 0.156$

$$RRA = -\frac{u"(x)}{u'(x)}.x = -\frac{-1/x^2}{1/x}.x = 1$$

The coefficient of relative risk aversion has the same value irrespective of the value of x.

2. You just inherited a treasure chest from your eccentric uncle, but don't know how to open it. Two methods suggest themselves. The first method costs \$20 and has a 10% chance of success. the second method costs \$400 and has a 80% chance of success. Which method should you try first? What is reduction in expected value if you reverse the order?

   Assume that the treasure is worth \$x.



Note: Numbers in parentheses are probabilities.

   I. Expected payoff for trying method 1 first:

   $$EV(M1, M2) = 0.1(x - 20) + 0.9[0.8(x - 420) + 0.2(-420)]$$

   $$EV(M1, M2) = 0.82x - 380$$

   II. Expected payoff for trying Method 2 first:

   $$EV(M2, M1) = 0.8(x - 400) + 0.2[0.1(x - 420) + 0.9(-420)]$$

   $$EV(M2, M1) = 0.82x - 404$$

   We can see that trying method 1 first gives a higher expected payoff. If we reverse the order, the expected payoff falls by \$24.

   Of course, one can also quit (not shown on tree) at any time. It is worth trying method 2 only if $x \geq 400/.8 = 500$, and worth trying method 1 only if $x \geq 20/.10 = 200$.

3. Consider the two player game described by the following payoff bimatrix.

	a	b	c
A	2, 0	1, 1	4, 2
B	3, 4	1, 2	2, 3
C	1, 3	0, 2	3, 0

a. Does either player have a dominant strategy?

Let's check for player 1:

Player 1 plays A, player 2 plays c

Player 1 plays B, player 2 plays a

Player 1 plays C, player 2 plays a

Now let's check for player 2:

Player 2 plays a, player 1 plays B

Player 2 plays b, player 1 plays either A or B

Player 2 plays c, player 1 plays A

We can see that no player has a dominant strategy which she can play irrespective of what strategies other player plays.

b. Does either player have a dominated strategy?

For player 1, strategy C is dominated by strategy A.

c. Which strategy profile survives iterated deletion of strictly dominated strategies? Explain each deletion very briefly?

For player 1, C is dominated by A, therefore C is never a best response for her.

Removing C from consideration, we find that for player 2, b is now dominated by c. Therefore player 2 will not play b and we can remove b from consideration.

We are left with:

	a	c
A	2, 0	4, 2
B	3, 4	2, 3

d. Find all Nash equilibrium (NE) in pure strategies, if any.

	a	c
A	2, 0	$\underline{4}$, $\underline{2}$
B	$\underline{3}$, $\underline{4}$	2, 3

Underlining RS's, we see that (A,c) and (B,a) are pure strategy NE.

e. Find all Nash equilibria (NE) in mixed strategies, if any.

Let's assume that player 1 plays A with probability p (call it σ_1) and player 2 plays a with probability q (call it σ_2).

Player 2 will mix such that: $Eu(A, \sigma_2) = Eu(B, \sigma_2)$

$\implies \qquad 2q + 4(1-q) = 3q + 2(1-q) \implies q = \frac{2}{3}$

Similarly, player 1 will mix to make player 2 indifferent between her two strategies:

$Eu(a, \sigma_1) = Eu(c, \sigma_1)$

$\implies \qquad 0p + 2(1-p) = 4p + 3(1-p) \implies p = \frac{1}{3}$

Therefore { (A, a) ; $(\frac{1}{3}, \frac{2}{3})$} is a mixed strategy NE.

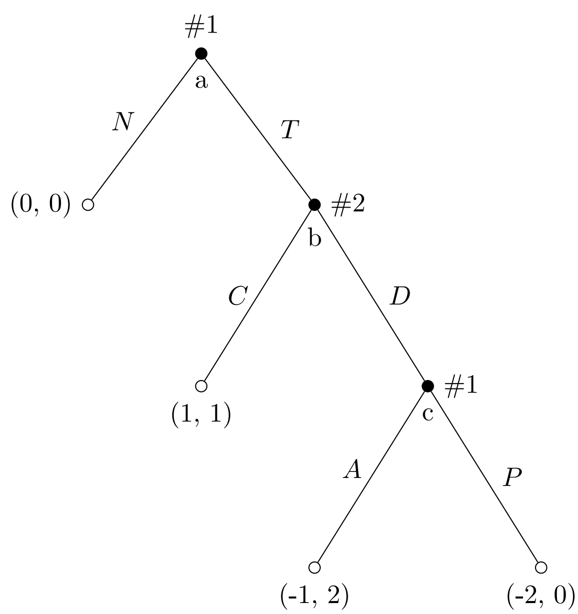
f. Find all payoff dominant NE and all risk dominant NE, if any.

No payoff dominant NE: player 2 does better in (B,a), player 1 does better in (A,c), and neither does better in the mixed NE, as you can check by evaluating the indifference conditions at the NE mixes.

(A,c) is risk dominant because both players are still BR-ing to tremble rates of up to 0.5.

4. Player #1 first chooses whether to trust (T) or not (N). If he chooses N then both players get payoff 0. Otherwise, player #2 then chooses whether to cooperate (C) or to Defect (D). If she chooses C then each of the players get payoff 1. Otherwise, #1 has the final move and chooses either accept (A) in which case the payoffs are -1 to #1 and 2 to #2, or else chooses punish (P) in which case the payoffs are -2 to #1 and 0 to #2.

a. Draw the extensive form game for this situation.



b. Find the subgame perfect Nash equilibrium (SPNE).

There are three subgames starting at nodes a, b and c.

Player #1 will choose A at node c $\Longrightarrow$ player #2 will choose D $\Longrightarrow$ player #1 will then choose N at node a.

$\Longrightarrow$ (NA, D) is a SPNE.

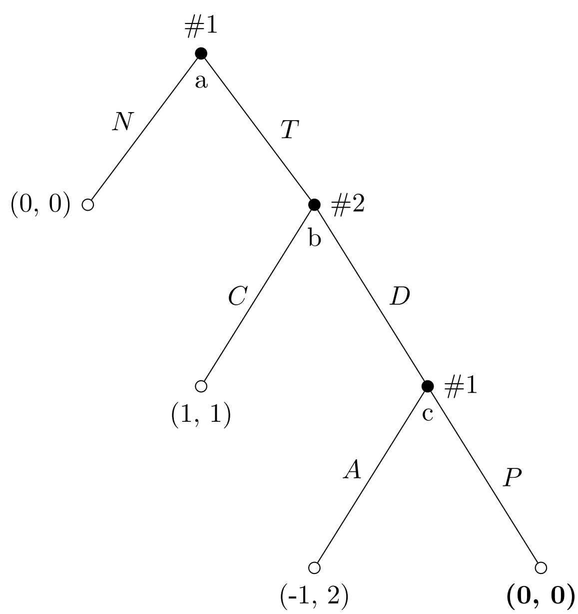
c. Find all other Nash equilibrium.

	C	D
NA	0, 0	0, 0
NP	0, 0	0, 0
TA	1, 1	-1, 2
TP	1, 1	-2, 0

Pure Strategy NE: {(NA,D),(NP,D), (TP, C)}.

Mixed NE include arbitrary mixes of NA and NP against D.

d. Now consider a different payoff for player#1 whose vengeful preferences give him payoff 0 (instead of -2) following P; the game is otherwise unchanged. Find the SPNE.



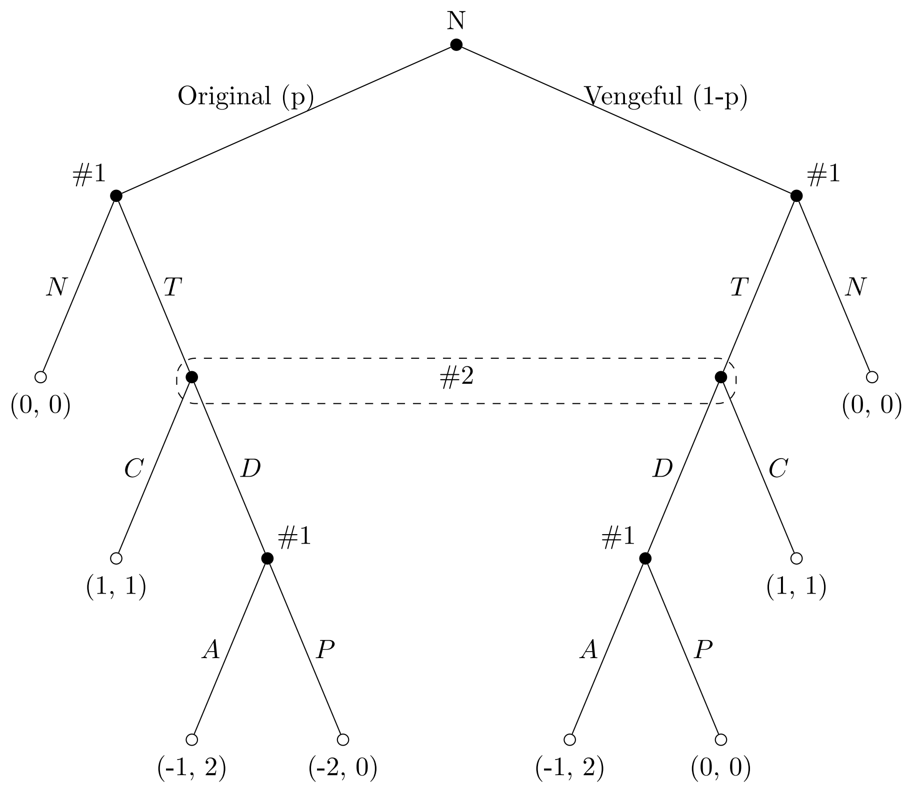
Again there are three subgames starting at nodes a, b and c.

Now player #1 will choose P at node c (instead of A earlier)

$\Longrightarrow$ player #2 will choose C $\Longrightarrow$ player #1 will choose T at node a.

$\Longrightarrow$ {TP, C} is SPNE.

e. Using the Harsanyi method, draw the game of incomplete information in which player #2 is unsure whether player #1 is vengeful as in part d, or is as in the original game.



f. For extra credit, find the range of probabilities (i.e., of player #2's initial beliefs about whether player #1 is vengeful) for which player #1 chooses T in BNE.

Let's assume player #2 believes player #1 is vengeful (like in d) with probability p.

Expected payoff for player #1 if chooses: N= $0.(1-p)+0.p=0$

if chooses T= $-1.(1-p)+1.p=2p-1$

Given the player #2's beliefs, player #1 will choose T only if $2p-1>0, \implies p>\frac{1}{2}$