

Midterm - ECON 200 - Fall 2016

October 25, 2016

1 Alkoids

Supply of alkoids is well approximated over the relevant range by the expression $p - 3$, and demand is well approximated by $12 - 2p$, where p is price.

1.1 What is the competitive equilibrium (CE) price p^* and quantity Q^* ?

Match supply and demand:

$$\begin{aligned}Q_S^*(p^*) &= Q_D^*(p^*) \\p^* - 3 &= 12 - 2 \cdot p^* \\3p^* &= 15 \\p^* &= 5 \\Q^*(5) &= 5 - 3 = 2\end{aligned}$$

1.2 What are the (own price) supply and demand elasticities at CE?

Own price elasticity is given by:

$$\begin{aligned}\varepsilon_D &= \frac{\partial D}{\partial P} \cdot \frac{P}{D} = -2 \cdot \frac{5}{2} = -5 \\ \varepsilon_S &= \frac{\partial S}{\partial P} \cdot \frac{P}{S} = 1 \cdot \frac{5}{2} = 2.5\end{aligned}$$

1.3 Suppose that a tax of $t = 0.6$ is now imposed. What is the tax revenue? How much comes from consumers? From producers? What is the deadweight loss?

Note that

$$\begin{aligned}Q_S^t(p_S) &= Q_D^t(p_D) = Q_D^t(p_S + t) \\p_S^t - 3 &= 12 - 2 \cdot (p_S^t + t) \\3p_S^t &= 15 - 2 \cdot 0.6 \\p_S^t &= 5 - 0.4 = 4.6 \\p_D^t &= 4.6 + 0.6 = 5.2 \\Q^t &= p_S^t - 3 = 4.6 - 3 = 1.6\end{aligned}$$

Tax revenue is

$$Q^t \cdot t = 1.6 \cdot 0.6 = 0.96$$

Suppliers receive $5 - 4.6 = 0.4$ less per unit, while demanders pay $5.2 - 5 = 0.2$ more. Hence suppliers pay $\frac{.4}{.6} = \frac{2}{3}$ of the burden and consumers the remaining $\frac{1}{3}$. The deadweight loss is

$$\begin{aligned} DWL &= \frac{1}{2} \cdot (p_D^t - p_S^t) \cdot (Q^* - Q^t) \\ &= \frac{1}{2} \cdot (5.2 - 4.6) \cdot (2 - 1.6) \\ &= 0.12 \end{aligned}$$

2 Cost Function

Suppose that you estimate the following equation:

$$\ln[c/y] = 2.1 + 0.6 \ln w_1 + 0.5 \ln w_2 \quad (1)$$

where \ln is natural log, c =total cost, y =output quantity, and the w_i 's are prices of key inputs.

2.1 What, if anything, can you say about the marginal cost function implied by eq. (1)?

Note that:

$$\begin{aligned} \ln c &= 2.1 + 0.6 \ln w_1 + 0.5 \ln w_2 + \ln y \\ c(w_1, w_2, y) &= e^{2.1} w_1^{0.6} w_2^{0.5} y \\ MC(w_1, w_2) &= e^{2.1} w_1^{0.6} w_2^{0.5} = \text{constant} \\ \therefore AVC(w_1, w_2) &= \text{constant} \\ &\therefore \text{CRS production} \end{aligned}$$

That is, MC is constant (independent of output level y , tho dependent on input prices of course), and so is AVC.

2.2 What can you say about returns to scale of the underlying production function?

As noted above, constant marginal cost implies constant average variable cost, which implies constant returns to scale in production.

2.3 Is the cost function homogeneous of any degree? Should it be homogeneous of some degree k (and if so, what k)? Comment very briefly.

A cost function should be homogenous of degree 1 in input prices, here w_1 and w_2 . We can check:

$$\begin{aligned} &e^{2.1} (Tw_1)^{0.6} (Tw_2)^{0.5} y \\ &e^{2.1} T^{0.6} w_1^{0.6} T^{0.5} w_2^{0.5} y \\ &T^{1.1} e^{2.1} w_1^{0.6} w_2^{0.5} y = T^{1.1} c \end{aligned}$$

It is homogenous of degree 1.1. Close, but not quite a proper cost function. Maybe there was a rounding error, or the appropriate regression constraint was not imposed.

2.4 (d) Write down the conditional demand function for input 1 implied by equation (1).

By Shephard's Lemma:

$$\begin{aligned}x_1^* &= \frac{\partial c(\mathbf{w}, y)}{\partial w_1} \\ &= 0.6 \cdot e^{2.1} w_1^{-0.4} w_2^{0.5} y \\ &= \frac{0.6}{w_1} \cdot c(\mathbf{w}, y)\end{aligned}$$

3 Abe's preferences

Abe's preferences can be represented by $u(x_1, x_2) = \ln x_1 + 2 \ln x_2$.

3.1 Which of the following utility functions (if any) also represent his preferences?

1. $v(x_1, x_2) = x_1 + 2x_2$
No, constant MRS, unlike u
2. $w(x_1, x_2) = x_1^{0.4} x_2^{0.8}$
Yes, note:

$$\begin{aligned}\ln w &= 0.4 \ln x_1 + 0.8 \ln x_2 \\ 2.5 \ln w &= \ln x_1 + 2 \ln x_2 \\ 2.5 \ln w &= u\end{aligned}$$

w is just a positive monotonic transformation of u . Alternatively, you could compute the MRS functions and see that they are the same.

3. $U(x_1, x_2) = x_1 + g(x_2)$.
No, $MU_1 = 1$, unlike u

3.2 Abe is currently consuming the bundle $(x_1, x_2) = (2, 3)$. How many extra microunits of good 2 would he require to just compensate him for the loss of a microunit of good 1?

This is just a question about the Marginal Rate of Substitution

$$\begin{aligned}MRS &= \frac{MU_1}{MU_2} = \frac{u_1}{u_2} \\ &= \frac{1/x_1}{2/x_2} \\ &= \frac{1}{2} \cdot \frac{x_2}{x_1} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}\end{aligned}$$

Abe would need 3/4 of a microunit of good 2 to compensate him.

3.3 What fraction of income m would you predict that Abe would spend on good 2 if, sometime later, prices were $(p_1, p_2) = (1, 4)$?

Solve for the optimal choice at this set of prices:

$$\begin{aligned}\frac{MU_1}{MU_2} &= \frac{p_1}{p_2} \\ \frac{1}{2} \cdot \frac{x_2}{x_1} &= \frac{1}{4} \\ x_2^* &= \frac{1}{2}x_1^*\end{aligned}$$

Plug this into the budget:

$$\begin{aligned}m &= p_1x_1 + p_2x_2 \\ m &= x_1^* + 4 \cdot \frac{1}{2} \cdot x_1^* \\ m &= 3x_1^* \\ x_1^* &= \frac{1}{3}m\end{aligned}$$

Abe will spend one-third of his income on good 1 and two-thirds on good 2.

4 Supply

Suppose that that a firm's cost function (for fixed input prices) is $c(y) = y^3 - 6y^2 + 10y + 8$.

4.1 What is its fixed cost? Variable cost? Average variable cost? Marginal cost?

- Fixed cost is 8.
- Variable cost is $y^3 - 6y^2 + 10y$
- Average variable cost is $\frac{c}{y} = y^2 - 6y + 10$
- Marginal cost is $\frac{\partial c}{\partial y} = 3y^2 - 12y + 10$

4.2 What is the firm's supply function $y(p)$? Its profit as a function of output price p ?

First find minimum efficient scale:

$$\begin{aligned}MC &= AVC \\ 3y^2 - 12y + 10 &= y^2 - 6y + 10 \\ 2y^2 - 6y &= 0 \\ 2 \cdot y \cdot (y - 3) &= 0\end{aligned}$$

The positive root is $y = 3$, so that is the minimum efficient scale, above which supply equals:

$$\begin{aligned}p &= MC \\ p &= 3y(p)^2 - 12y(p) + 10 \\ 0 &= 3y(p)^2 - 12y(p) + 10 - p\end{aligned}$$

Solving via the quadratic formula

$$\begin{aligned} y(p) &= \frac{1}{6} \cdot \left(12 + \sqrt{12^2 - 4 \cdot 3 \cdot (10 - p)} \right) \\ &= 2 + \frac{1}{6} \sqrt{144 - 120 + 12p} \\ &= 2 + \frac{1}{3} \sqrt{6 + 3p} \end{aligned}$$

It can only be the positive portion of the parabola, otherwise output falls with rising prices. This would not be optimizing. We need to find p at minimum efficient scale:

$$\begin{aligned} MC(3) &= 3(3)^2 - 12 \cdot 3 + 10 \\ &= 27 - 36 + 10 = 1 \end{aligned}$$

Therefore:

$$y(p) = \begin{cases} 2 + \frac{1}{3} \sqrt{6 + 3p} & \text{if } p \geq 1 \\ 0 & \text{if } p < 1 \end{cases}$$

Profit as a function of price:

$$\begin{aligned} \pi(p) &= p \cdot y(p) - y(p)^3 + 6 \cdot y(p)^2 - 10 \cdot y(p) - 8 \text{ for } p \geq 1 \\ &= (10 - p) \cdot \left(2 + \frac{1}{3} \sqrt{6 + 3p} \right) - \left(2 + \frac{1}{3} \sqrt{6 + 3p} \right)^3 + 6 \cdot \left(2 + \frac{1}{3} \sqrt{6 + 3p} \right)^2 - 8 \text{ for } p \geq 1 \\ \pi(p) &= \begin{cases} (10 - p) \cdot \left(2 + \frac{1}{3} \sqrt{6 + 3p} \right) - \left(2 + \frac{1}{3} \sqrt{6 + 3p} \right)^3 + 6 \cdot \left(2 + \frac{1}{3} \sqrt{6 + 3p} \right)^2 & \text{for } p \geq 1 \\ -8 & \text{for } p < 1 \end{cases} \end{aligned}$$

5 HFCs

Hydrofluorocarbons (HFCs) are currently the main coolant used in air conditioners, but earlier this month a world-wide treaty was signed to ban HFCs starting in 2019. Suppose that current total output is $Q^o = 100$ at price $p^o = 200$ and that using a new sort of coolant instead of HFCs will raise marginal cost for air conditioners by about 20.

5.1 What other information would you need to estimate the impact on price and quantity sold?

Demand, or at least own-price demand elasticity ε_D . We also need to know supply elasticity ε_S . It seems safe to assume that demand is competitive, and we will assume (unless we learn otherwise) that supply is reasonably competitive, so we can apply the CE model.

5.2 b

Write down a very rough numerical guess for each necessary piece of information (e.g., if income elasticity of demand for air conditioners were important, you might write down = 1.0) and compute the predicted impact on price and quantity.

Answer: Let us assume

$$\begin{aligned} \varepsilon_D &= -3 \\ \varepsilon_S &= 1 \end{aligned}$$

One approach is to notice that the MC increase works very much like a per-unit tax. It causes supply to shift up (or in):

$$S'(P) = S(P - 20)$$

Using the formula in the next part of the question:

$$\Delta P \approx 20 \cdot \frac{\varepsilon_S}{\varepsilon_S + |\varepsilon_D|} = 20 \cdot \frac{1}{4} = 5$$
$$\Delta Q \approx 20 \cdot \frac{|\varepsilon_D| \varepsilon_S}{\varepsilon_S + |\varepsilon_D|} \cdot \frac{Q_0}{P_0} = 20 \cdot \frac{3}{4} \cdot \frac{100}{200} = 7.5.$$

Another approach is geometric. Just use the elasticities to draw linear approximations to S and D curves, shift the S curve up by 20, and compute the impact using slopes and intersections of old and new lines.

5.3 Replace your numerical guesses by algebraic symbols (e.g., just use the symbol in the example above) and write your prediction as a function of the other information.

$$\Delta P \approx 20 \cdot \frac{\varepsilon_S}{\varepsilon_S + |\varepsilon_D|}$$
$$\Delta Q \approx 20 \cdot \frac{|\varepsilon_D| \varepsilon_S}{\varepsilon_S + |\varepsilon_D|} \cdot \frac{100}{200} = 10 \cdot \frac{|\varepsilon_D| \varepsilon_S}{\varepsilon_S + |\varepsilon_D|}$$