ECON 204B PS4

Answer Key - Pedro, Roberto and Thinkling

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I Problems

I.1 Question 0

(from previous PS) Two countries, A and B, are negotiating on trade. Relative to status quo, the sum of possible gains for the two countries is 10. The gains can be split in any fashion without affecting the total.

(a) *Calculate the Nash Bargaining Solution for this trade game.*

In this question, we assume that single country will gain 0 if not trade with others. And all the countries are risk neutral. Due to the symmetry of the two parties, the Nash Bargaining solution is divide the gain evenly and get (5,5) as the utility vector.

(b) *A third country C enters the trade negotiations. It can provide* ***mutual gains*** *of 15 with country A, or 5 with country B, or net gains of 5 (in addition to the 10 already available) with A and B jointly. Write out the characteristic function for this three-player game.*

In this question, we assume that single country will gain 0 if not trade with others. And all the countries are risk neutral.

Thus, the characteristic function for this three-player game is a mapping:

$$\nu(\varnothing) = \nu(A) = \nu(B) = \nu(C) = 0$$
$$\nu(A, B) = 10, \nu(B, C) = 5, \nu(A, C) = 15$$
$$\nu(A, B, C) = 15$$

(c) *Is this game convex?*

According to the definition, the game is convex iff:

$$\nu(S) + \nu(T) \leq \nu(S \cap T) + \nu(S \cup T), \forall S, T \subset N$$

Let's check from the smaller sets to larger ones. Because $\nu(\varnothing) = \nu(A) = \nu(B) = \nu(C) = 0$, the inequality holds for all singleton sets.

Then, take sets $\{A, C\}$ and $\{B, A\}$,

$$\nu(A, C) + \nu(B, A) = 25 \geq \nu(A, B, C) + \nu(A) = 15$$

This contradicts the definition. Thus, the game is not convex.

(d) *Calculate the core of the three-player game.* From the previous questions:

$$\begin{aligned}
&\nu(\varnothing) = \nu(A) = \nu(B) = \nu(C) = 0 \\
&\nu(A, B) = 10 \implies \nu(C) \leq 5 \\
&\nu(B, C) = 5 \implies \nu(A) \leq 10 \\
&\nu(A, C) = 15 \implies \nu(B) \leq 0 \\
&\nu(A, B, C) = 15
\end{aligned}$$

Thus, the core is the singleton $(A, B, C) = (10, 0, 5)$

(e) *Calculate the Shapley value of the three player game.*

Shapley value			
ρ	MC_1	MC_2	MC_3
ABC	0	10	5
ACB	0	0	15
BAC	10	0	5
BCA	10	0	5
CAB	15	0	0
CBA	10	5	0
Σ	45	15	30
ϕ	7.5	2.5	5

In Table 1, because we have $3! = 6$ permutations, we need to divide the summation of each column by 6 to get the Shapley Value.

I.2 Question 1

There are two risk neutral players, A and B. Nature chooses an amount of money, x, from a uniform distribution over [0, 1000], and places it in an envelope for A. Nature then tosses a fair coin. If it is heads, Nature places y = 2x in an envelope for B. If it's tails, Nature places y = x/2 in an envelope for B. The players (who know that a coin was tossed but not how it came out) look at their envelopes in private, and then announce "trade" or "no trade" simultaneously. Trade occurs if and only if both announce "trade".

(a) *Is there an equilibrium in which trade occurs with positive probability? [Hint: to get started, think about whether there are high values of x or y that preclude trade. You might want to use the logic of iterated dominance.]*

If B gets an amount of money $y \geq 500$, he will know that $y = 2x$, i.e., the probability that $y = x/2$ is zero, so he will definitely choose not to trade.

Knowing this, A will choose not to trade if $x \geq 250$, because trade is advantageous for her only if $y = 2x \geq 500$, in which case (as we have seen) B will choose not to trade.

Anticipating that A will not trade unless $x < 250$, B should say "no trade" unless $y < 125$, since only in this case could B's envelope contain $x/2$ enabling trade to B's advantage.

Iterating such reasoning, the only situation in which both players are willing to trade is $(x = 0, y = 0)$ and in this case, the trade can occur with positive probability.

(b) *A philosophy student tells you that he has heard of this problem, and insists that trade will occur with probability 1. His reasoning is that each player will look at the amount z in her envelope, compute her post-trade value as* $0.5(z/2) + 0.5(2z) = 1.25z > z$, *and so regard trade as personally beneficial; indeed, ex ante, the trading opportunity is mutually beneficial, which any economist should understand. What, if anything, is wrong with the philosophy student's reasoning?*

This reasoning uses prior probabilities, before the players open their envelopes or think about when the other player is willing to trade. After the players open their envelopes, they use this message to update their expected payoffs, which will be different than $1.25z$. Thus, the philosophy student should have considered the updating of probabilities as foreseen by Bayes' theorem.

I.3 Question 2

An uninformed player U possesses an object that is either type a (and worth \$120 to him) or else type b (and worth \$60 to him). U doesn't know the true type and regards them as equally likely. By contrast, the other player I does know the true type but values type a at \$100 and type b at \$200. Consider a one-shot game in which I bids a chosen amount x, and U either accepts or rejects that bid. Then the true type is revealed to U and the players receive their final payoffs. In your analysis of this game, assume that x_k(denoting I's bid when the commodity is of type k = a, b) is an integer, and confine your attention to pure strategy Perfect Bayesian Equilibria. Recall that a PBE includes both a strategy profile and beliefs at all information sets, including those never reached in equilibrium. When the same outcome is generated by different PBEs, you need only specify one of the PBEs fully.

(a) *Find all the PBE outcomes in which* $x_a = x_b$. *Explain why you think you have found all the pooling PBE outcomes.*

Assume (as is standard) that both players are risk neutral, and know the structure of the game, as diagrammed in Figure 1. Payoff vectors are for player I and U respectively.

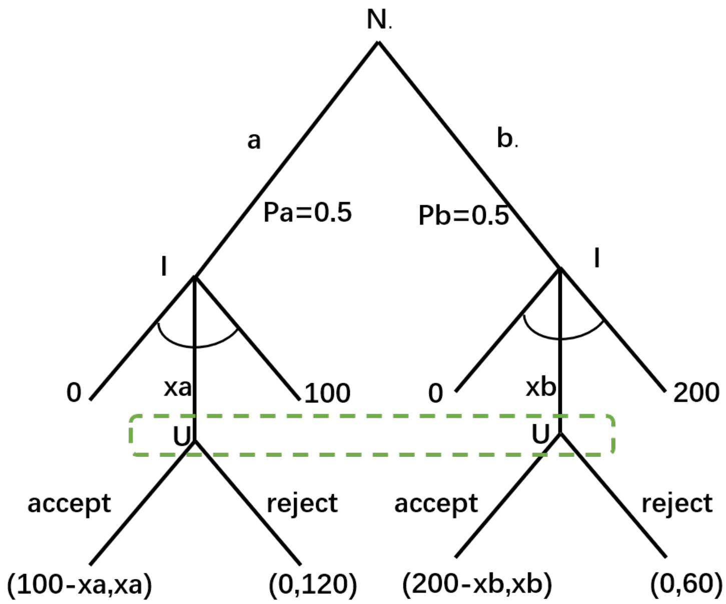

Figure 1: Game Tree for problem 2

We know that if player I sees type a, he will bid less than 100. If he has the strategy $x_a = x_b = s$, we know that $x \leq 100$.

If $x_a = x_b = x$, then player U can not have additional information to update his prior probability, so his expected payoffs are

$$\pi_{Accept} = 0.5x + 0.5x = x$$
$$\pi_{Reject} = 0.5 \times 120 + 0.5 \times 60 = 90$$

Thus, player U will accept if $x \geq 90$.

Therefore, if the bid is $90 \leq x \leq 100$, it is accepted. The pooling PBE outcomes are : $(90 \leq x \leq 100, A)$. The problem doesn't ask for it, but the other parts of pooling PBEs include beliefs $\mu = 1/2$ and rejection when (conterfactually) $90 < x$.

(b) *Find all the PBE outcomes in which $x_a \neq x_b$ and x_a is rejected and x_b is accepted. Again, explain why you have found all such separating PBE outcomes. Are there any PBE in which x_a is accepted and x_b is rejected?*

U will reject $x_a < 120$, and I will not offer $x_a > 100$. Likewise, U will reject $x_b < 60$, and I will not rationally offer $x_b > 200$. To get a separating PBE, therefore, we must have $0 \leq x_a \leq 100 \leq x_b \leq 200$.

In PBE, player U understands this and assigns beliefs accordingly. Seeing a bid $x < 100$, he infers that it is type a, and rejects the bid. Seeing a bid $x \in (100, 200)$, he infers type b, and accepts.

Therefore, the equilibria are:

For x_a $(x_a \leq 100, R, \mu_u(a|x_a) = 1)$ For x_b $(100 < x_b \leq 200, A, \mu_u(b|x_b) = 1)$

(c) *Assume that I does not play weakly dominated strategies and U knows this. This assumption puts a restriction on out-of-equilibrium beliefs: when U sees an offer that is weakly dominated for one type but not for the other, U should believe with probability 1 it's the other type. Which of the separating PBE found in part b satisfy this assumption?*

If player I plays $x_b \in (101, 200)$, it is weakly dominated: he could have bought the object at a lower price whenever the type turns out to be b.

Thus a refined PBE is $(\{x_a \leq 100, x_b = 101\}$, Reject iff $x \leq 100, \mu(a|x \leq 100) = 1, \mu(a|x > 100) = 0)$.

I.4 Question 3

Slug Insurance (SI) is planning to sell policies to 20,000 UCSC and Cabrillo students. 80% of the students are low risk with average health costs (measured in thousands of dollars per year) CL = 1 and standard deviation = 1. The other 20% are high risk with average health costs CH = 2 and standard deviation = 4. All students have risk aversion coefficient r = 0.2 in utility function $u(X) = EX - \frac{1}{2}rVar(X)$. A risk-neutral for-profit company, SI has negligible overhead costs. It can't identify the risk type of individual students but does know the 20% − 80% breakdown.

(a) *Compute the willingness to pay for health insurance by each type of student, L and H.*

$$u_L(X) = EX - \frac{1}{2}rVar(X) = -1 - 1/2 * 0.2 = -1.1$$

$$u_H(X) = EX - \frac{1}{2}rVar(X) = -2 - 1/2 * 0.2 * 16 = -3.6$$

Thus, the willingness to pay is 1.1 for L and 3.6 for H.

(b) *What annual premium (P*) would allow SI to break even if all students joined?*

The premium equals the expectation of the cost for one year:

$$P* = 0.8 * 1 + 0.2 * 2 = 1.2$$

(c) *SI charges 0.4 (or $400) above the break-even point. At that premium, which students would find it worthwhile to join SI?*

SI charges 1.6 for each group, which is higher than the willingness to pay for L. Thus, only high risk students would find it worthwhile to join SI.

(d) *What are SI's profits in case c above? How can SI adjust their strategy to increase profits? How much could they make?*

If the premium is 1.6 (and only high risk students join), the profits in thousands are given by:

$\pi = 4 * 1.6 - 4 * 2 = -1.6,$
where 4 thousand is the number of high risk students. To make higher profits, the company would have to discriminate between high risk and low risk students. That is not observable but, if somehow the company could completely overcome that problem, it would be able to make the following profits:

Low risk : premium of 1.1 $\pi_L = 16 * 1.1 - 16 * 1 = 1.6$

High risk: premium of 3.6 $\pi_H = 4 * 3.6 - 4 * 2 = 6.4$

So the highest conceivable profit is $8,000 using complete discrimination, which is not possible. What the company can do to discriminate prices is to adjust the premium and the deductible to attract low risk or high risk students. Naturally, if the premium is high but deductible is low, the students with higher risk will be attracted to the offer. Conversely, if the premium is low but deductible is high, those with lower risk will be attracted. Ideal choices of these insurance plans might result in profits that are positive but less than 8 thousand.

I.5 Question 4

Professor P is hiring a teaching assistant, Mr A. Professor P's payoff function is $x - s$, where x is the number of hours A works and s is the amount she pays A. Mr A's payoff function is s–$x^2/2$; he gets payoff 0 if he doesn't work for P.

(a) *What choices of x and s will maximize P's utility (subject, of course, to the constraint that A is willing to work for her)?*

$$\max_{s,x} \pi_p = x - s$$
$$s.t.\ s - x^2/2 \geq 0$$
$$L = x - s + \lambda(s - x^2/2)$$
$$\partial L/\partial s = -1 + \lambda = 0$$
$$\partial L/\partial x = 1 - \lambda x = 0$$
$$\Rightarrow x = 1 \;\&\; s = 1/2$$

Thus, we know P will maximize her utility at $(x = 1, s = 1/2)$

(b) *Suppose that P offers a wage schedule s(x) = ax + b, where A picks x. What choices of a and b will now maximize her utility?*

$$\max_{x} \pi_a = s(x) - x^2/2 = ax + b - x^2/2$$
$$\partial \pi_a/\partial x = a - x = 0$$
$$\Rightarrow x = a$$

Thus, professor always knows that the student will choose $x = a$ as long as $s - x^2/2 \geq 0$.

Thus, the professor's problem now becomes:

$$\max_{a,b} \pi_p = a - a^2 - b$$
$$s.t.\ a^2 + b - a^2/2 \geq 0$$
$$L = a - a^2 - b + \lambda(a^2 + b - a^2/2)$$
$$\partial L/\partial a = 1 - 2a + \lambda a = 0$$
$$\partial L/\partial b = -1 + \lambda = 0$$
$$\Rightarrow a = 1 \ \&\ b = -1/2$$
$$\Rightarrow x^* = 1, s^* = 1/2$$

Thus, the professor should adopt the function$s = x - 1/2$ to get the highest payoff.

(c) *Could P obtain higher utility using a non-linear wage schedule?*

No, because linear wage schedule(b) has already get the highest payoff in(a), no other schedule could generate higher payoff.

I.6 Question 5

Consider the following two player game. Player 1 (the child) moves first, and takes an action A≥ 0 that produces income Ic(A) for himself, and income Ip(A) for the other player (the parent). The parent observes Ic(A) and Ip(A) and then chooses a bequest B to leave to the child. The child's utility is V = Ic(A) + B. The parent's utility from her own consumption is U = Ip(A) - B, but she also cares about the utility of the child. The parent maximizes the strictly increasing, strictly concave, and twice continuously differentiable welfare function W(U, V). The bequest B can be either positive or negative. Prove the Rotten Kid Theorem: In SPNE, the child chooses the action that maximizes the family's aggregate income, Ic(A) + Ip(A), even though only he has selfish preferences. Is this the first-best outcome from the parent's perspective?

By the parent's first-order condition,

$$\frac{\partial W}{\partial U}\frac{\partial U}{\partial B}(A,B) + \frac{\partial W}{\partial V}\frac{\partial V}{\partial B}(A,B) = 0$$

But from the definitions, we see that $\frac{\partial U}{\partial B}(A,B) = -1 = -\frac{\partial V}{\partial B}(A,B)$. Thus,

$$\frac{\partial W}{\partial U} - \frac{\partial W}{\partial V} = 0$$

This partial differential equation is solved by any function $W(U+V) = W(I_C(A) + I_P(A))$. Since W is increasing, we see that the parent's goal is to maximize family income $I_C(A) + I_P(A)$. In Nash equilibrium, the parents cannot gain anything by changing unilaterally their strategy. The child could only ensure this if he/she maximizes $W(I_C(A) + I_P(A))$, which means he/she also should maximize $I_C(A) + I_P(A)$, given that W is strictly increasing.

Remarks from Dan.

- A more detailed argument looks at the FOC for the child and differentiates the parent's FOC wrt A. Straightforward but messy algebra reveals that the child also satisfies the FOC for maximizing family $I_C(A) + I_P(A)$.
- Like the previous problem, this is a variant on the Principal/Agent problem without the subtleties of incentive constraints and risk aversion.

II Textbook problems.

Write up and turn in your solutions to MCWG problems 12.B.1, 12.B.3, 12.B.6, 12.D.4, 12.E.4, 13.B.3, 13.C.4, and 14.B.3; for extra credit do 14.C.8.

II.1 12.B.1

(a) Prove that

$$\frac{(p^m - c'(q^m))}{p^m} = \frac{1}{\epsilon_p}$$

where

$$\epsilon_p = \lim_{\Delta p^m \to 0} \frac{-\Delta q(p^m)}{\Delta p^m} * \frac{p^m}{q(p^m)}$$

We can write ϵ_p as

$$\epsilon_p = -q'(p^m) * \frac{p^m}{q(p^m)}$$

The monopolist problem and FOC are

$$\max_p p * q(p) - c(q(p)) \tag{1}$$

$$q(p^m) + p^m * q'(p^m) = c'(q(p^m)) * q'(p^m) \tag{2}$$

Rearranging this, we get:

$$-q(p^m) = [p^m - c'(q(p^m))] * q'(p^m) \tag{3}$$

$$\frac{-q(p^m)}{q'(p^m) * p^m} = \frac{p^m - c'(q(p^m))}{p^m} = 1/\epsilon_p \tag{4}$$

Which completes the proof.

(b) Given

$$\frac{1}{\epsilon_p} = \frac{(p^m - c'(q^m))}{p^m}$$

And given the fact that

$$\frac{(p^m - c'(q^m))}{p^m} < \frac{p^m - 0}{p^m}$$

Then

$$\frac{1}{\epsilon_p} < 1$$

and

$$\epsilon_p > 1$$

II.2 12.B.3

The monopolist is maximizing $x(p,\theta)p - c(q,\phi)$:

$$\max_p x(p,\theta)p - c(x(p,\theta),\phi) \tag{5}$$

$$F.O.C. \quad \underbrace{x_1 p + x - c_1 x_1}_{f} = 0 \tag{6}$$

Differentiating f with respect to θ and ϕ:

$$\frac{\partial f}{\partial p} = 2x_1 + px_{11} - (c_{11}x_1^2 + c_1 x_{11}) \tag{7}$$

$$\frac{\partial f}{\partial \theta} = x_{12}(p - c_1) + x_2(1 - x_1 c_{11}) \tag{8}$$

$$\frac{\partial f}{\partial \phi} = -c_{12}x_1 \tag{9}$$

Thus, by implicit function theorem, we can get:

$$\frac{\partial p}{\partial \theta} = -\frac{\frac{\partial f}{\partial \theta}}{\frac{\partial f}{\partial p}} = -\frac{x_{12}p + x_2 - c_{11}x_1x_2 - c_1x_{12}}{2x_1 + px_{11} - (c_{11}x_1^2 + c_1x_{11})} \tag{10}$$

$$\frac{\partial p}{\partial \phi} = -\frac{\frac{\partial f}{\partial \phi}}{\frac{\partial f}{\partial p}} = \frac{c_{12}x_1}{2x_1 + px_{11} - (c_{11}x_1^2 + c_1x_{11})} \tag{11}$$

Assuming that the partial derivatives satisfy $x_1 < 0$, $x_{11} < 0$, $c_1 > 0$, and $c_{11} > 0$, if we want $\frac{\partial p}{\partial \theta} > 0$, we need to have $x_{12} > 0$ and $x_2 > 0$; if we want $\frac{\partial p}{\partial \phi} > 0$, we need to have $c_{12} > 0$.

II.3 12.B.6

Suppose the gov tax t on the monopolist:

$$\max_q p(q)q - c(q) - tq \tag{12}$$

$$F.O.C\ p'q + p - c' - t = 0 \tag{13}$$

If we want the monopolist to act efficiently, i.e. the Lerner index equals to 0, which means the marginal cost equals the price, we have to make sure that $p'q = t$. Also because we know $p' < 0$, it implies that t should be negative and gov. should always gives subsidy instead of tax.

II.4 12.D.4

infinitely repeated Bertrand oligopoly with $\delta \in [0.5, 1)$.

(a) The most profitable price is the monopoly price. Take $c_1 > c_2$. In monopoly setting, $(p_1 - c_1)q(p_1) > (p_2 - c_1)q(p_2)$ and $(p_2 - c_2)q(p_2) > (p_1 - c_2)q(p_1) \Rightarrow (c_1 - c_2)(q_2 - q_1) > 0$ This implies that $q_2 > q_1 \Rightarrow p_1 > p_2$. Thus, π decreases and p increases if c increases.

(b) Suppose c_1 increases to c_2 at period 2, the future payoff will fall which leads to the lower sustained profits in period one and beyond.

The collusion equilibrium starting from period 2 gives each of n cartel members payoff π_2/n in every following period. Unilaterally defecting in period 1 gives the defector the entire monopoly profit π_1. Therefore the cartel is viable only if

$$\pi_1 \leq \pi_1/n + [\delta/(1-\delta)]\pi_2/n \implies \delta \geq \frac{(n-1)\pi_1}{\pi_2 + (n-1)\pi_1}.$$

If oligopolists are not exceptionally patient and the cost increase is substantial, then π_2 will be small relative to $(n-1)\pi_1$ and the cartel can not be sustained at the monopoly price. Lowering the price in period 1 can lower the temptation π_1 to defect sufficiently to sustain the cartel.

II.5 12.E.4

If the planner cannot control the perfect cartel behavior, the price and industry output will not be changed no matter how many firms enters. However, the aggregate fixed cost nK will increases as the firm number increases, which indicate that the optimal number will be only one firm.

If the planner cannot control entry, the number of firms will be $n \approx \pi^m/K$. This means that the entered firm will equally separate the monopoly profit without any benefit to consumers.

II.6 13.B.3

(a) *show that the more capable workers are the ones choosing to work at any given wage.*

Suppose firms offer a wage of w. All workers of type θ, with r(θ) $\geq$ w, will accept the wage and work. Suppose there exists a worker θ' with r(θ') = w.

Then because r is decreasing, all workers of type $\theta \geq \theta' = w$ will work. Thus, the more capable workers are the ones who will work at any given wage.

(b) *Show that if $r(\theta) > \theta, \forall\theta$, the result is Pareto efficient.*

Suppose firms offer the wage of $\bar{\theta}$, the highest θ. Since the minimal required wage is $r(\bar{\theta}) > \bar{\theta}$, no workers of any type will work. Firms employ nobody in this equilibrium, but it is Pareto efficient because they are all more productive at their reservation activity — that is what $r(\theta) > \theta$ means.

(c) *Suppose that $\exists \theta, s.t.\ r(\theta) < \theta for \theta > \bar{\theta}$ and $r(\theta) > \theta for \theta < \bar{\theta}$. Show that any competitive equilibrium with strictly positive employment necessarily involves too much employment relative to the Pareto optimal allocation of workers.*

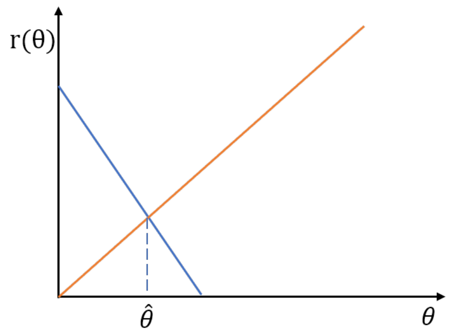

Figure 2: r(θ) and θ

If w = $\hat{\theta}$, only workers of type $\theta \geq \hat{\theta}$ will accept the wage w and work. But the firms will demand more workers than there are in supply, thus the market is not clear.

If $w < \hat{\theta}$, only workers of type $\theta \geq \theta_w$, where $r(\theta_w) = w$ will accept the wage w and work. The market is still not clearing because $E(\theta|\theta \geq \theta_w) > w$.

Thus, firms will consider only $w > \hat{\theta}$. Then some workers of type $\theta < \hat{\theta}$ will accept the job, even though it would be more efficient for them to stick with their reservation activity instead.

Dan comments that this problem is unrealistic in that selection is the opposite of adverse, but it gives practice in thinking thru how competitive equilibrium works when there is self selection (participation constraints).

II.7 13.C.4

The workers' utility maximization problem is the following.

$$\max_e w(e) - c(e,\theta) = w(e) - e^2/\theta$$
$$F.O.C:\ w'(e) - 2e/\theta = 0$$

Since the firms are competitive, w(e^*) = θ in equilibrium. Therefore, $w'(e) = 2e/w(e)$. By solving this differential equation by direct integration, or by guess and check, we can confirm that $w(e) = \sqrt{2}e$. So the unique separating PBE is $(e = \theta/\sqrt{2}, w(e) = \sqrt{2}e)$. The belief here is $\mu(\theta = \sqrt{2}e|e) = 1$.

II.8 14.B.3

Manager: $EU = E[w] - \phi\, Var(w) - g(e), g'(0) = 0, \lim_{e\to\infty} g'(e) = \infty$, $\pi|e \sim N(e, \sigma^2)$

(a) *linear compensation schemes:* $w(\pi) = \alpha + \beta\pi$, *show that* $E(U|w(\pi), e, \sigma^2) = \alpha + \beta e - \phi\beta^2\sigma^2 - g(e)$

$$\begin{aligned} E(U|w(\pi), e, \sigma^2) &= E[w(\pi)] - \phi\, Var(w(\pi)) - g(e) \\ &= \alpha + \beta E(\pi) - \phi\beta^2 Var(\pi) - g(e) \\ &= \alpha + \beta e - \phi\beta^2\sigma^2 - g(e) \end{aligned}$$

(b) *Derive the optimal contract when e is observable*

If we can observe the effort, the optimal contract for the employer will allow the wage to exactly compensate for the effort function,i.e.w is a function of $(\pi|e)$. Therefore, taken the wage as given, the problem for manager will become:

$$\max_e EU = E[w(\pi|e)] - \phi\, Var(w(\pi|e)) - g(e)$$
$$F.O.C\ g'(e^*) = w'(e^*)$$

Suppose the compensation function for employer is $w = e$, then the optimal e^* is given by $g'(e^*) = 1$.

(c) *Derive the optimal linear compensation scheme when e is not observable. What effects do changes in* β *and* σ^2 *have?*

If the linear compensation scheme is $w(\pi) = \alpha + \beta\pi$.

First, consider a given effort level e', we know the $\alpha + \beta e' - \phi\beta^2\sigma^2 - g(e')$ will reach the maximum by part (a); then, we know from part (b), $g'(e') = w'(e') = \beta$.

Second, substitute β with $g'(e')$, the problem becomes:

$$\max_e EU = \alpha + g'(e')e' - \phi g'(e')^2\sigma^2 - g(e')$$
$$F.O.C\ g''(e')e' + g'(e') - 2\phi g''(e')\sigma^2 - g'(e') = 0$$

Then, we can use differential equation formula to solve for g'(e).

II.9 14.C.8

(a) The indifference curves of the two types and the firm's isoprofit curve are depicted in figure 3.

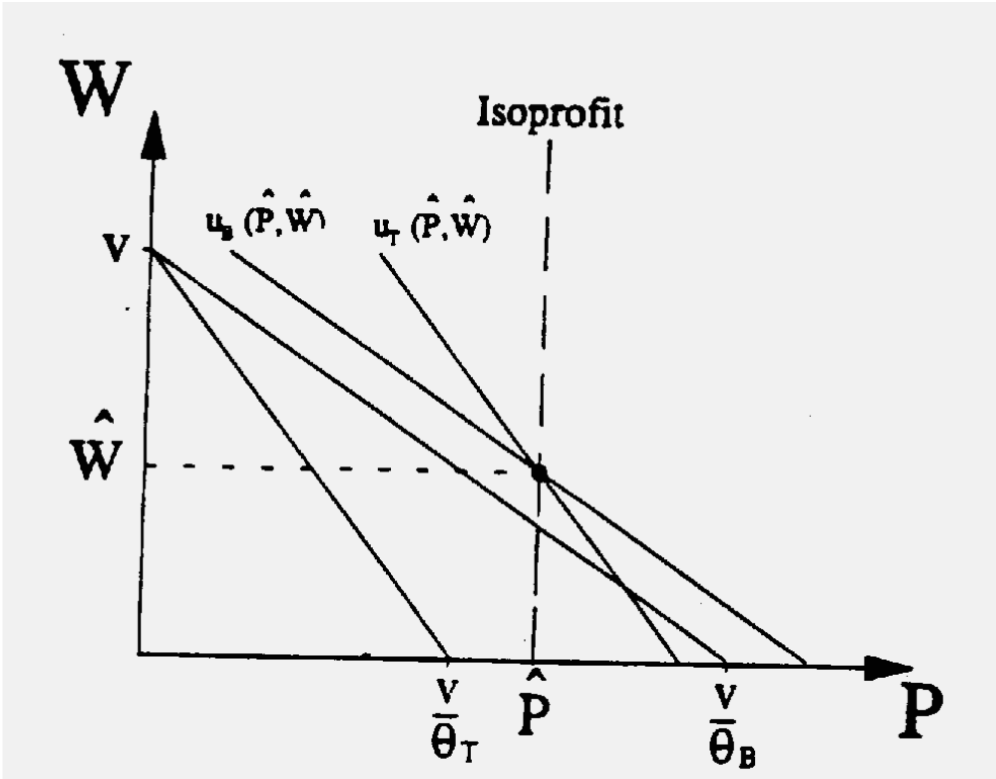

Figure 3: The indifference curves of the two types and the firm's isoprofit curve

The problem that Shangri La would want to solve is:

$$\max_{P_T,P_B,W_T,W_B} \lambda P_T + (1-\lambda)P_B \tag{14}$$

$$s.t. \theta_T P_T + W_T \leq v \tag{15}$$

$$\theta_B P_B + W_B \leq v \tag{16}$$

$$\theta_B P_B + W_B \leq \theta_T P_B + W_B \tag{17}$$

$$\theta_B P_B + W_B \leq \theta_B P_T + W_T \tag{18}$$

$$P_B, W_B, P_T, W_T \geq 0 \tag{19}$$

(b) In part (a), constraint 15 and 18 together with $\theta_B < \theta_T$, imply that constraint 16 is redundant, so it is never binding. Constraint 15 therefore must bind for, if it would not, we can reduce P_T and P_B by $\epsilon > 0$, and the remaining constraints will still be satisfied. Thus, tourists will be indifferent between buying and not buying a ticket.

(c) Assume that $\{(P_T, W_T), (P_B, W_B)\}$ is an optimal, incentive compatible contract, and assume in negation that $W_B > 0$. Now reduce W_B by $\epsilon > 0$, and increase P_B by $\frac{\epsilon}{\theta_b}$ so that B type's utility does not change, and the firm earns higher profits from the B type. We need to check that the T type will not choose this compensation package. Indeed,

$$\theta_T P_T + W_T \leq \theta_T P_B + W_B = \theta_T(P_B + \frac{\epsilon}{\theta_T}) + (W_B - \epsilon) < \theta_T(P_B + \frac{\epsilon}{\theta_B}) + (W_B - \epsilon),$$

contradicting that $\{(P_T, W_T), (P_B, W_B)\}$is an optimal incentive compatible contract. Therefore, we must have $W_B = 0$. If in an optimal contract, the business travelers were not indifferent between (P_T, W_T) and (P_B, W_B), we could slightly raise P_B and all the constraints would remain satisfied (recall that 16 is redundant), and the firm would earn higher profits from the business types. Therefore, in an optimal contract, we must have the business types indifferent between (P_T, W_T) and (P_B, W_B).

(d) The trade that the firm faces is: By lowering P_T and increasing W_T so as to keep the tourists indifferent between buying a ticket or not, the firm can increase P_B (recall that $W_B = 0$). From parts b and c above, we can conclude that if the firm raises W_T by ϵ and lowers P_T by $\frac{\epsilon}{\theta_T}$ so that the tourists remain indifferent between buying or not, then to keep the business types indifferent between their package and the new tourist package, it can increase P_B by $\frac{\epsilon(\theta_T - \theta_B))}{\theta_T \theta_B}$. Since this trade-off is linear, it is true no matter where we are in the (P,W) space, and therefore it will be profitable if and only if

$$\lambda \frac{\epsilon}{\theta_T} < (1-\lambda)\frac{\epsilon(\theta_T - \theta_B)}{\theta_T \theta_B},$$

or

$$\frac{\lambda}{1-\lambda} < \frac{\theta_T - \theta_B}{\theta_B}$$

Note that this is independent of the cost c, because this is a revenue trade-off (the costs are the same for both types). Therefore, the price discrimination scheme can take on two forms:

(1) If $\frac{\lambda}{1-\lambda} < \frac{\theta_T - \theta_B}{\theta_B}$, then only the high types will be served and the scheme will be (this assumes that $c < \frac{v}{\theta_B}$)

$\{(\mathrm{P}_T, W_T), (P_B, W_B)\}$= (0,v),(v$\overline{\theta_B,0)}$

(2) If $\frac{\lambda}{1-\lambda} > \frac{\theta_T - \theta_B}{\theta_B}$, then both types will be served and the scheme will be a pooling scheme with (this assumes that $c < \frac{v}{\theta_T}$)

$$(P_T, W_T) = (P_B, W_B) = (\frac{v}{\theta_T}, 0).$$

Since the direction of the inequality in the condition above determines the type of scheme, it is easy to see how changes in θ, θ_T and θ_B will affect the scheme: If the proportion of B types is large enough (λ small enough), the firm will choose to serve only the B types. If the B types suffer less from prices (θ_B is smaller), then the firm is more likely to serve only them. If the T types suffer less from prices (θ_T is smaller), then the firm is more likely to serve them as well as the B types. Changes in the cost c are discussed in part (e) below.

(e) As long as $c < \frac{v}{\theta_B}$, and we are in case (1) as described in part (d) above, the firm will decide to serve only the business types. If however, we are in case (2) above, and $\frac{v}{\theta_T} < c < \frac{v}{\theta_B}$, then the scheme described in (d) above cannot be optimal because the firm is losing money. In such a case, the firm will choose the scheme described in case (1) of part (d), and serve only the business types. If $c > \frac{v}{\theta_B}$, the firm will choose not to operate at all.