Cycles and Instability in a Rock-Paper-Scissors Population Game[1]

Tim Cason, Dan Friedman, Ed Hopkins

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[1] Review of Economic Studies 81:1, pp. 112-136 (2014).

Motivation

- Rock-Paper-Scissors, aka RoShamBo, Shoushiling (China) or Jan-ken-pon (Japan) is played everywhere to resolve 2-player disputes. It may date back to the Han Dynasty 2000 years ago.
- Iconic for game theorists, especially evolutionary game theorists:
  - Simplest example of intransitive dominance: Rock $\prec$ Paper $\prec$ Sissors $\prec$ Rock ...
  - Hence should yield cyclic dynamics in population games.
  - Really? do they persist? do they converge to a limit point or limit cycles or ...?
- These issues recur in more complex theoretical settings, and in applications (e.g., lizards, voting, Edgeworth pricing games, ..).

The research questions

In simple RPS-like matrix games:

1. Under what conditions does play converge to the unique interior NE? Or to some other interior profile?
2. Under what conditions do we observe cycles?
3. If cycles persist, does the amplitude converge to a maximal, minimal, or intermediate level?

The overarching question:
When do we need to go beyond equilibrium theory to analyze strategic interaction?

Focus: stable vs unstable payoff matrix

$\mathbf{S} = \begin{pmatrix} 36 & 24 & 66 \\ 96 & 36 & 30 \\ 24 & 96 & 36 \end{pmatrix}$ has unique NE (0.25, 0.25, 0.5) w/ payoff 48.

$\mathbf{U_a} = \begin{pmatrix} 60 & 0 & 66 \\ 72 & 60 & 30 \\ 0 & 72 & 60 \end{pmatrix}$ NE is still (1,1,2)/4 with payoff 48

Performance variable: population distribution over time.

Period averages, distance from NE, Cycle rotation index.

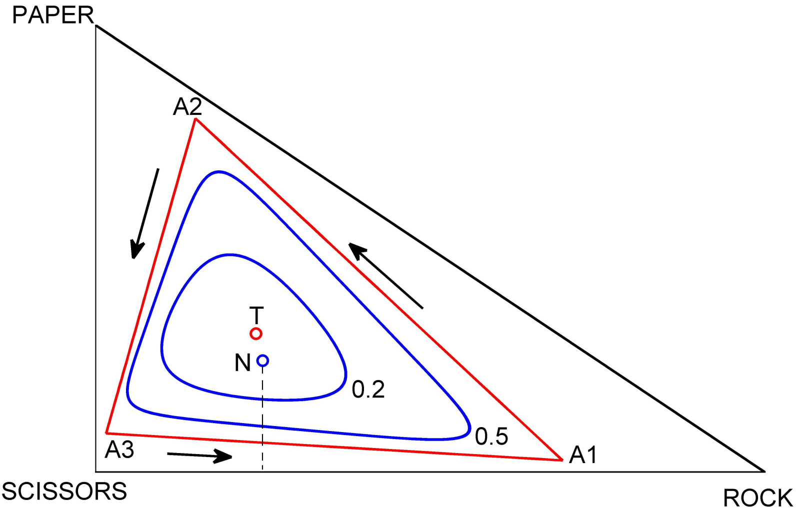


Figure: The Shapley triangle $A_1A_2A_3$ for game U_a with the TASP (T) and the Nash equilibrium (N). Also illustrated are orbits for the perturbed best response dynamics for precision parameter values 0.2 and 0.5.

Testable Hypotheses

1. **Nash Equilibrium (NE):** average play will be at the NE $(0.25, 0.25, 0.5)$ and average payoff will be 48 in all treatments.
2. **BR Dynamics:**
    1. In S, there will be counter-clockwise cycles that diminish in amplitude over time with ultimate convergence to NE.
    2. In U_a, there will be persistent counter-clockwise cycles that approach the Shapley triangle limit cycle.
    3. In U_b, there will be persistent clockwise cycles that approach the Shapley triangle limit cycle.
    4. Thus the average distance from NE will be consistently higher in U_a and U_b than in S.

Our treatments

- Two different asymmetric RPS matrices mentioned earlier, S and U_a, both with same NE which $\neq$ centroid and (for U_a) $\neq$ TASP.
- Action set treatments: Discrete pure, discrete mixed, continuous slow, continuous instant.
- 2x4, plus bonus treatment, reversed matrix (U_b) with CS.
- ConG1.0 for 11 sessions each w/8 Ss matched RR for 5 blocks of 5 180-sec periods. Discrete has 20 subperiods of 9 sec each.
- Balanced incomplete block design, split between Purdue and UCSC.

Thus **treated nuisances** are: site, action set.
Constant nuisances: length of periods, matrix entries, ...
Randomize for: sequence effects, learning, boredom, ...

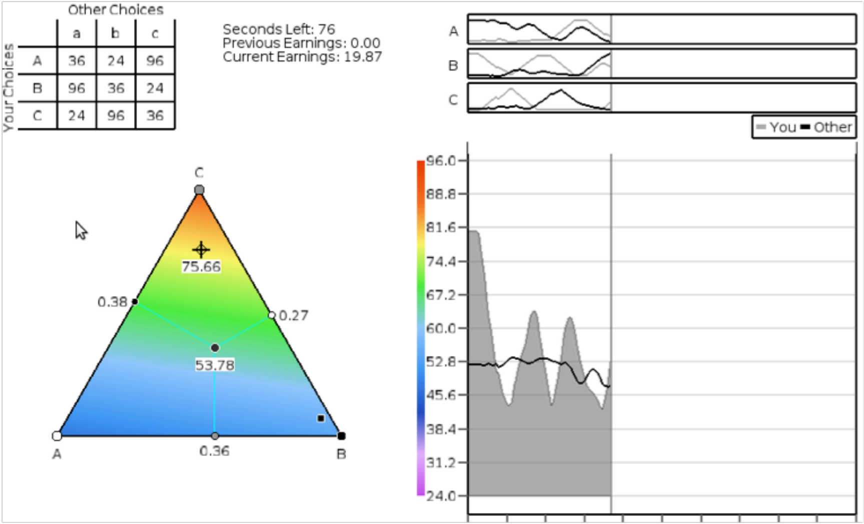
Design details

	Block 1	Block 2	Block 3	Block 4	Block 5
Sess D1	U3-DM	S3-CI	U3-DP	S3-DM	U3b-CS
Sess D2	U3b-CS	U3-CS	S3-CS	U3-CI	S3-DP
Sess D3	S3-CS	U3-DM	S3-CI	U3b-CS	S3-DM
Sess D4	U3-CI	S3-DM	U3-DM	S3-CS	U3-CI
Sess D5	S3-DP	U3b-CS	U3-DP	S3-CI	U3-CS
Sess D6	U3-CS	S3-DP	U3-CI	S3-DM	U3-DP
Sess D7	S3-CI	U3-CS	U3b-CS	S3-CS	U3-DM
Sess D8	U3-DP	S3-DM	U3-DM	S3-DP	U3-CI
Sess D9	S3-CI	U3-DP	S3-DP	U3-CS	S3-CS
Sess D10	S3-DM	U3-CI	S3-CS	U3-DP	S3-CI
Sess D11	U3-CI	S3-DP	U3-CS	U3b-CS	U3-DM

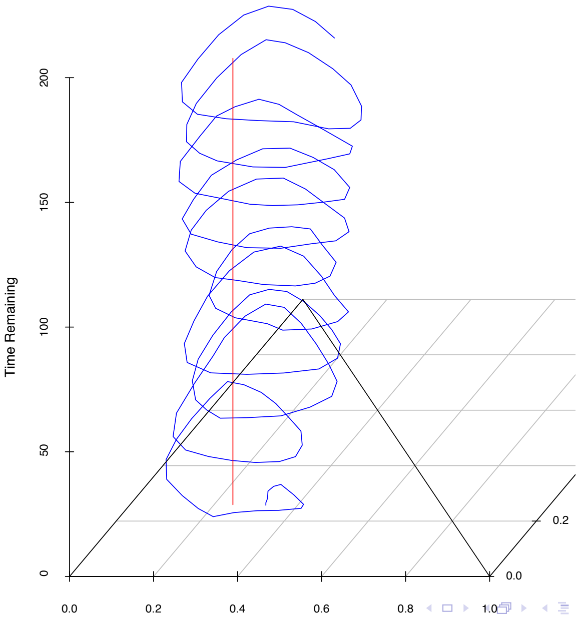
Balanced Incomplete Block Design

Every treatment appears in Blocks 1 and 5, at least 8 out of 9 treatments appear in Blocks 2 through 4, and no treatment appears more than two times in any one block.
This design also achieves our goals of not repeating any treatment within a session (so we get 6 independent observations for each), and the matrix changes every block.

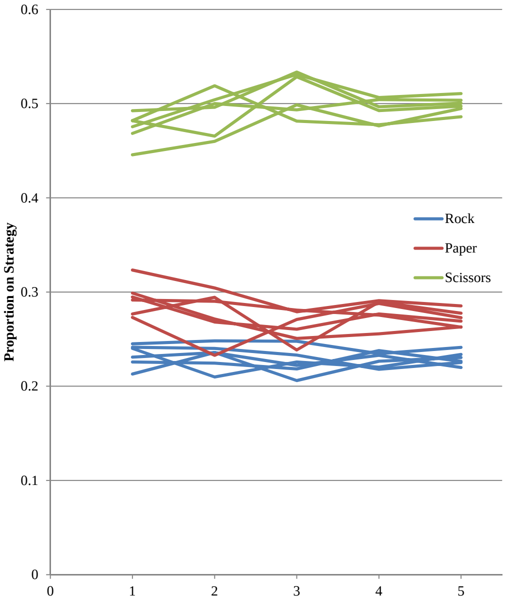
ConG Software: CS treatment (10 sec transit)



Session 2, period 6: U_a matrix, Continuous-Slow.



Mean choice by period within block: S, Contin-Instant.



Time average behavior and tests

Game-Condition	Rock	Paper	Scissors	Payoff
Nash Equilibrium	0.25	0.25	0.5	48
TASP (Ua)	0.242	0.31	0.449	51.1
TASP (Ub)	0.31	0.242	0.449	51.1
S Continuous-Instant	0.226^{N}	0.269^{N}	0.504	47.59^{N}
S Continuous-Slow	0.236^{N}	0.265	0.500	48.03
S Discrete-Mixed	0.242	0.294^{N}	0.464	47.95
S Discrete-Pure	0.247	0.320^{N}	0.433^{N}	47.57^{N}
Ua Continuous-Instant	0.247	0.318^{N}	0.435^{N}	49.82^{NT}
Ua Continuous-Slow	0.228^{NT}	0.281^{N}	0.491^{T}	49.08^{NT}
Ua Discrete-Mixed	0.225	0.342^{NT}	0.433^{N}	49.70^{NT}
Ua Discrete-Pure	0.205^{N}	0.337^{N}	0.458^{N}	50.71^{N}
Ub Continuous-Slow	0.303^{N}	0.240	0.457^{N}	48.81^{NT}

Cycle rotation index

A new statistic invented this summer.

- Tripwire = Poincare section = dashed line in Fig 1 from NE to edge.
- CCT = Count of left-to-right crossings, CT=# right-to-left.
- $CRI = \frac{CCT-CT}{CCT+CT} \in [-1, 1]$.
- Thus $CRI \approx 1 (\approx -1)$ indicates consistent c-cw (cw) cycles, and $CRI \approx 0$ indicates no consistent cycles.

Table: Mean Transits and Cycle Rotation Indexes

Game-Treatment	CCW	CW	*CRI*
S Continuous-Instant	24.1	5.8	0.64*
S Continuous-Slow	9.3	0.9	0.86*
S Discrete-Mixed	2.1	1.3	0.30
S Discrete-Pure	0.5	0.7	-0.04
U_a Continuous-Instant	30.3	1.9	0.89*
U_a Continuous-Slow	8.3	0.0	1.00*
U_a Discrete-Mixed	1.8	0.3	0.78*
U_a Discrete-Pure	0.9	0.2	0.68*
U_b Continuous-Slow	0.3	8.5	-0.94*

Note: * Denotes Index significantly (p-value $< 5\%$) different from 0 according to 2-tailed Wilcoxon test.

Summary

- EVG theory predicts cycles in RPS games, but they have not previously been detected.
- Cycles are prominent in the new continuous action treatments, most spectacularly for U_a -CS and U_b-CS.
- *CRI* shows direction of cycles is as predicted when they occur.
- Central tendency (TimeAverage) predicted rather well by TASP, though amplitude is less than ShapleyPolygon.
- S game partially vindicates prediction that cycle amplitude $\rightarrow 0$: amplitude is smaller than for U game, but settles in at positive value.
- U game seems to have limit cycles, as suggested by PBR, not heteroclinic cycle as predicted by replicator dynamics.

Conclusion and Speculation

- NE is not bad for predicting LR average in these games, but it is a rather poor SR predictor.
- EGT predicts far better in SR and TASP beats NE.
- Learning models and EGT more generally help us grasp "instability," an increasingly important theme for social scientists.