ECON 204B Topic on the graph of SOSD

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We know that the distribution F second order stochastically dominates G (SOSD) if

i $E(L) = E(M)$;

ii $\int_{-\infty}^{x} F(t)dt \leq \int_{-\infty}^{x} G(t)dt$

The conventional graph for SOSD is usually assumed that the interception point for $F(x)$ and $G(x)$ is the point where $E(L)$ equals $E(M)$(Figure1).

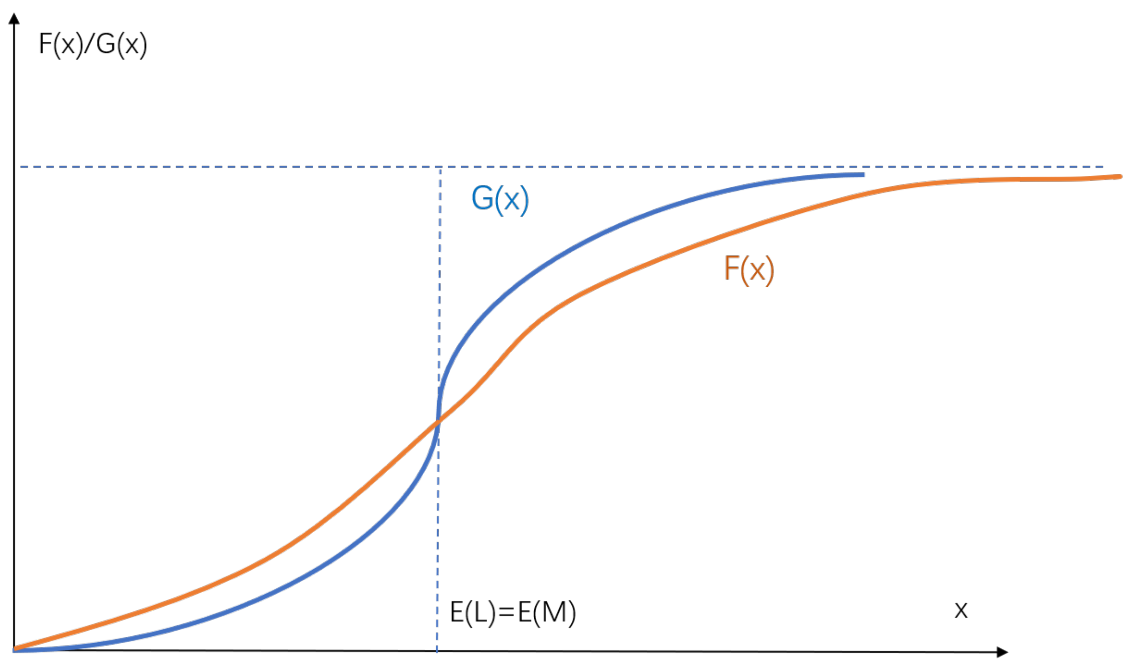

Figure 1: Conventional graph for SOSD

However, after carefully analyzing the polygon of the graph, we can find that the conventional figure is only a particular case, and the interception point(s) of CDF does not have the relation with expectation as we stated above.

To show this, we need to visualize the **two** conditions of SOSD on the CDF-graph. One interesting and critical question is how to find E(x)? We will use both area and point to express its value in the following steps.

Using integration by part:

$$\underbrace{\mathrm{E(L)}}_{\mathrm{H2}} = \int_0^b x dF(x) = \underbrace{b}_{H1+H2} - \underbrace{\int_0^b F(x)dx}_{\mathrm{H1}} \tag{1}$$

From Figure.2,We know that b is the area of the rectangle b*1(area H1+H2); $\int_0^b F(x)dx$ is the area among F(x), x=b and x-axis(area H1). As a result, E(x) is the area among F(x), y=1 and y-axis(area H2).

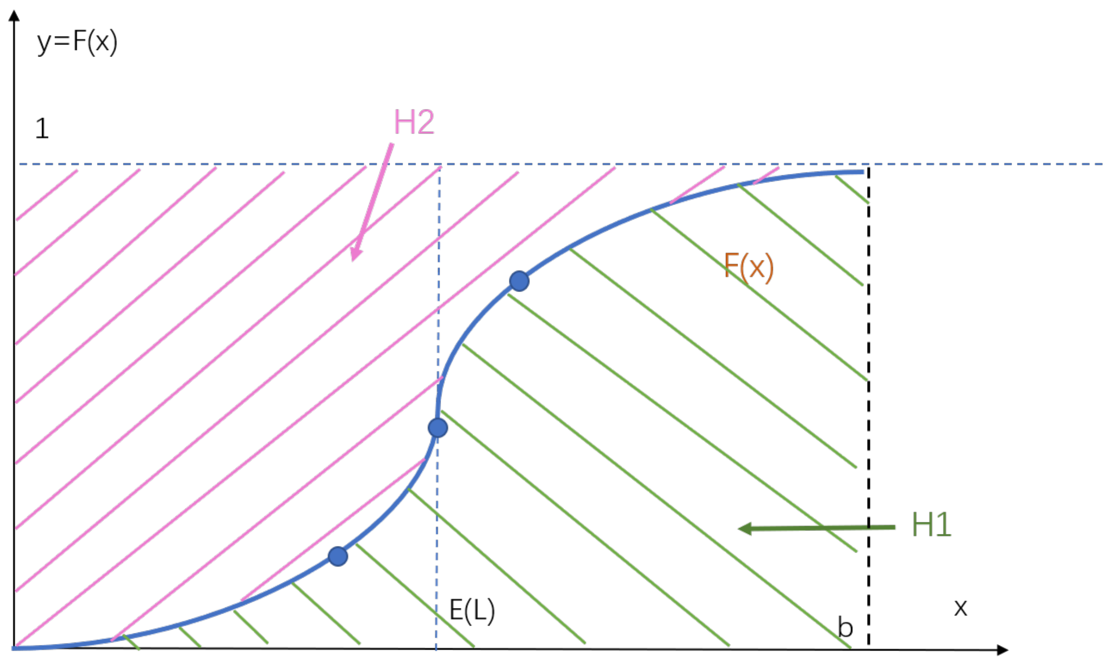

Figure 2: Express E(x) of F(x) as an area on the graph

After expressing E(x) as an area on the graph, we can transfer it to the x-axis after finding a vertical line making $T_1 = T_2$(Figure.3). In this case, the area between, $y = 1, y = 0, x = 0$ and the vertical line is still equals to H2, i.e. E(x). This transformation could imply that the vertical line and x-axis intercept at E(x). Denote this vertical line as area-complementary line.

The result above implies that if the area-complementary lines for F(x) and G(x) are the same, then $E(L) = E(M)$. And the shape or concavity of the functions does not impact our result.

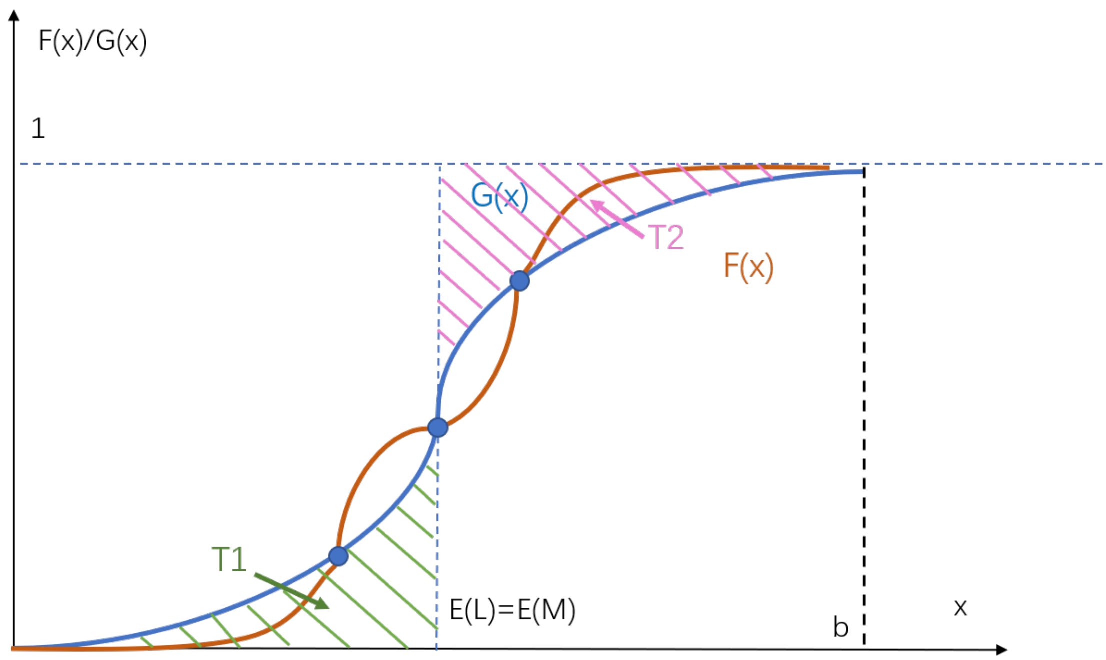

Figure 3: Express expectation on x-axis

Second, how to make sure that $\int_{-\infty}^{x} F(t)dt \leq \int_{-\infty}^{x} G(t)dt$? This condition states the area under F(x) should always be lower than G(x) at any x on the interval. For multi-interception cases(Figure.4), as long as $S1 \geq 0$ and $S2 \leq S1$ and $S3 \geq 0$ and $S1 + S3 \leq S2 + S4$, the second condition could be satisfied.

After visualizing two conditions, we can easily give a counter example. Two conditions could be easily checked for Figure 4. However, there are three interceptions and it implies that not all the interception points represents E(L)=E(M).

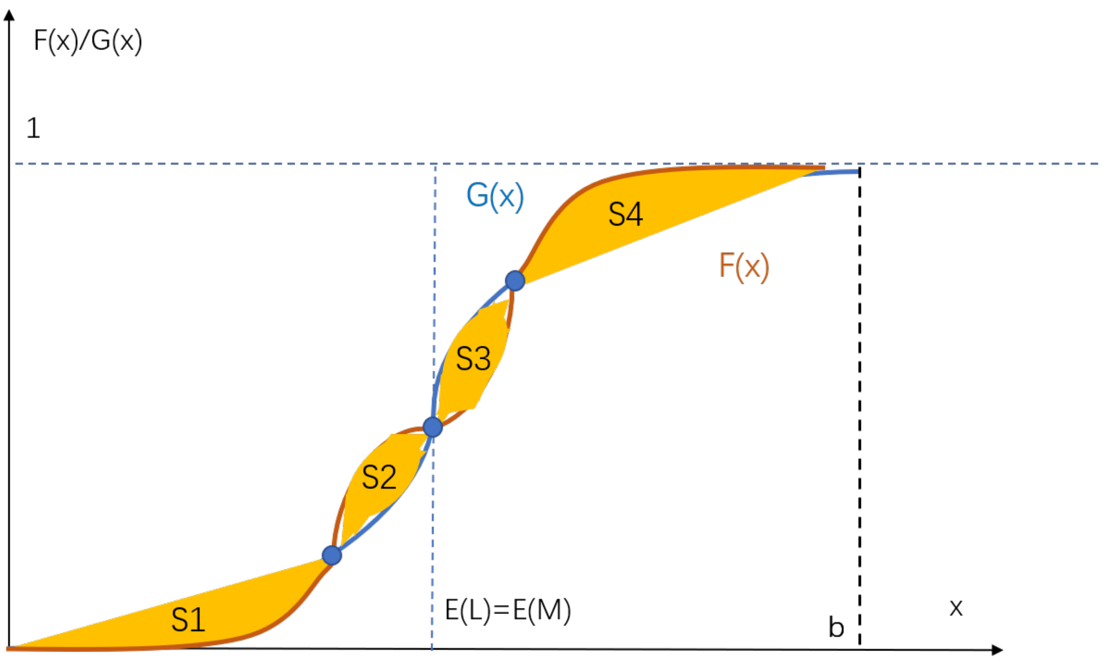

Figure 4: Compare integration of G(x) and F(x)

Moreover, after combining two fusiform S2 and S3 into one in Figure4, making sure $S2 + S3 < S1$ at the same time, the area-complementary line will hit F(x) and G(x) at a different point. It could further verify that the interception of F(x) and G(x) and E(L)(or E(M)) have no relation.