Problem Solution Set 3

Chapter 16

14) Technical progress is not neutral. Less labor was required to spin the same amount of cotton.

\[ \frac{MP_L}{MP_K} = (2.5)(0.64)A^{0.25}R^{-0.25} : 1.6(A/R)^{0.25} \]
\[ \frac{MP_K}{MP_L} = (2.5)(0.76)A^{0.35}R^{-0.34} : 1.875(A/R)^{0.35} \]

b) MRTS \( w \): \( -\frac{MP_L}{MP_K} = -\frac{(\frac{A}{L})(\frac{R}{K})}{(\frac{L}{K})(\frac{A}{R})} \) assuming \( A \) on x-axis.

MRTS \( p \): \( -\frac{MP_L}{MP_K} = - \frac{1}{3} (\frac{A}{R}) \)

(c) Yes consider:
\[ G_w = 3A^{0.25}R^{0.75} \text{ and } G_p = 25A^{0.25}R^{0.75} \]
These have same MRTS but different marginal products.

20) a) constant returns to scale
b) if \( \alpha + \beta = 1 \), constant
\[ \text{if } \alpha + \beta > 1, \text{ increasing} \]
\[ \text{if } \alpha + \beta < 1, \text{ decreasing} \]
\[ \text{e) same as above} \]
d) if \( d = 1 \), constant
\[ \text{if } d > 1, \text{ increasing} \]
extra credit

30) If \( f(xL, xW) = x^y f(L, K) \) then \( \frac{dF}{dxL} = x f'_1(xL, xW) = x^y f'_1(L, K) \)
and \( f'_1(xL, xW) = x^y f'_1(L, K) \) then \( \frac{dF}{dxW} \frac{f'_1(xL, xW)}{f'_2(xL, xW)} = f'_1(L, K) \)

31) From above with \( y_1, f'_1(xL, xW) = f'_1(L, K) \) and 
\( f'_2(xL, xW) = f'_2(L, K) \) which implies 
\( \frac{f'_1(xL, xW)}{f'_2(xL, xW)} = \frac{f'_1(L, K)}{f'_2(L, K)} \)

and the MRTS is independent of \( x \).

32) If \( f(xL, xW) = x^y f(L, K) \) then \( \frac{dF}{dx} \frac{f'_1(xL, xW)}{f'_2(xL, xW)} = \frac{f'_1(L, K)}{f'_2(L, K)} \)

\( \frac{f'_1(xL, xW)}{f'_2(xL, xW)} = \frac{f'_1(L, K)}{f'_2(L, K)} \)

Set \( x = 1 \): \( L f'_1(L, K) + K f'_2(L, K) = y f(L, K) \)

Chapter 7

1(c, d)

2) The firm choose labor-advance technology.

3) cost

\( \text{AC (linear mean)} \)

\( \text{AC (pure labor)} \)
23) \( AC(q) = 0.55q^{0.67} + 800q^{-2} \)

\[
\frac{dAC(q)}{dq} = 0.3685q^{-0.33} + 1600q^{-3} = 0
\]

\( q = 4.341.93 \)

\( q \approx 2.3 \)

with the tax

\( AC(q) = 0.55q^{0.67} + 800q^{-2} + 400q^{-1} \)

\[
\frac{dAC}{dq} = 0.3685q^{-0.33} - 1600q^{-3} - 400q^{-2} = 0
\]

\( q \approx 0.8 \)

27) a) You must set \( \frac{dAC}{dq} = 0 \) for each firm. The minimum point of \( AC \), is at \( q = 2 \). At plant 2, the min is 1.

b) The firm should produce 3 units in plant 1, and 1 unit in plant 2.

31) Suppose capital is fixed at \( \bar{K} \).

\( F = \bar{K} \)

\( VC = W = 10L \)

Total Cost: \( C = F + VC = 20K + 10L \)

\( q = 10L \)

\( 0.32 - 0.3q \)

\( L = (0.1q)^{3.125} \)

\( AVC = VC/q = 0.0075q^{-1.25} \)

Suppose \( \bar{K} = 1 \)

\( L = (0.1q)^{3.125} \)

\( VC = 10L = 0.0075q^{3.125} \)

\( AVC = VC/q = 0.0075q^{2.125} \)

\( MC = dV/dq = (0.0075q^{3.125})/dq = 0.023q^{2.125} \)
If it had to use same factor quantity as US (k = L = 100) c = 350 x 450 = 800

extra credit

37)