1. Suppose that Amy makes risky choices as if maximizing the expected value of the Bernoulli function $u(m) = \ln(m + 4)$. She is faced with a situation in which she will receive either $-3$ or $21$; the outcomes are equally likely.

a. what is the expected (mean) outcome? Variance of outcome? (4pts)

**Solution:** By definition, the expected value is

$$E[m] = \frac{1}{2}(-3) + \frac{1}{2}(21) = 9.$$ 

By definition, the variance is

$$V[m] = \frac{1}{2}(-3 - E[m])^2 + \frac{1}{2}(21 - E[m])^2 = 144.$$ 

b. what is Amy’s certainty equivalent of the risky outcome? What is the maximum amount she would be willing to pay an insurer to get the mean outcome for sure? (6pts)

**Solution:** By definition, certainty equivalent, $m^{CE}$, satisfies

$$u(m^{CE}) = EU(L),$$ 

$$\Leftrightarrow \ln(m^{CE} + 4) = \frac{1}{2}(-3 + 4) + \frac{1}{2}(21 + 4),$$ 

$$\Leftrightarrow m^{CE} + 4 = \exp(\ln 5) \Leftrightarrow m^{CE} = 1.$$ 

Risk Premium = $E[m] - m^{CE} = 9 - 1 = 8$ is maximum she is willing to pay.

c. What is Amy’s coefficient of absolute risk aversion at the mean outcome? Her coefficient of relative risk aversion? (4pts)
Solution: By definition,

\[ ARA(m) = -\frac{u''(m)}{w'(m)} = -\frac{-(m + 4)^2}{(m + 4)^{-1}} = \frac{1}{m + 4}. \]

\[ RRA(m) = -\frac{u''(m)m}{w'(m)} = -\frac{-(m + 4)^2 m}{(m + 4)^{-1}} = \frac{m}{m + 4}. \]

Evaluate these at mean outcome \( m = 9 \) yields \( \frac{1}{13}, \frac{9}{13} \) respectively.

2. Demand elasticity for Basic Economy airline tickets on the SFJ-TSK route is \(-3.0\), while elasticity is \(-2.0\) for Economy and \(-1.2\) for Business class. If the marginal cost is $100 for each seat, what price would be a profit-maximizing airline charge for each kind of ticket? How would your answer change if the airline improved Business Class so that the marginal cost became $120? (12pts)

Solution: By the formula based on the first-order condition of monopoly problems,

\[ p_{\text{Basic}}^* = \frac{|-3.0|}{|-3.0| - 1}(100) = 150 \]

\[ p_{\text{Economy}}^* = \frac{|-2.0|}{|-2.0| - 1}(100) = 200 \]

\[ p_{\text{Business}}^* = \frac{|-1.2|}{|-1.2| - 1}(100) = 600. \]

If the marginal cost becomes $120 for business class, \( p_{\text{Business}}^* = 720. \)

3. In 2004, China abolished its Agricultural Tax (AT), reducing funds to many county governments. For example, even after a new subsidy from the central government, one county government in Zhejiang province lost about 20% of its revenue due to the AT abolition. Assume that the AT revenues had to be spent entirely on the county’s agricultural programs, but other tax revenues can be spent on any of the county’s priorities (e.g., education, health pensions and administration) as well as on agricultural programs.

a. sketch the 2003 opportunity set (or budget set) for that county government with \( x \) = expenditure on agricultural programs and \( y \) = expenditures on non-agricultural
priorities. Then draw its 2004 opportunity set given that the AT abolition (plus subsidy) is the only change from 2003. (6pts)

**Solution:** See the figures. The figures ignore new off-setting subsidy introduced after the abolition of AT, but incorporating it is immediate and does not change implications (the opportunity set will be expanded outward slightly and y-intercept of the set will be slightly larger). The slope of the downward sloping part of the set is $-1$ before and after the abolition because both axes are measured in dollars.

Also, the figures assume an interior solution where the county spends strictly more than AT on agriculture.

b. Suppose that the county spent 30% of its total revenue on agricultural programs in 2003. What percent of its revenue do you predict that it would spend on agricultural programs in 2004? Please assume that the government maximizes an unchanging utility function $u(x, y)$ over its changing opportunity set. What properties of utility functions (e.g, monotonicity, etc.) does your prediction assume? (6pts)
**Solution:** If we assume an interior solution before the abolition and if utility function is **homothetic**, then since the slope of the opportunity set does not change before and after the abolition, then optimum must occur at the same ray from the origin. Thus, in this case, the expenditures on $x$ and $y$ shrink proportionately, but the fraction on agriculture remains 30%.

4. For the last several years, the only limo service from an isolated community to the nearest airport was provided by Suber. It experiences inverse demand $p = 200 - q$, and cost per ride of $c = 40$.

a. what price $p$ and number of rides $q$, will maximizes Suber’s profit.

**Solution:** Suber maximizes profit,

$$
\max_q (200 - q)q - 40q. 
$$

The first-order condition yields

$$
200 - 2q^* = 40.
$$

Thus, 

$$
q^* = 80,
$$

and $p^* = 200 - q^* = 120$.

b. What then are consumer surplus (CS), producer surplus (PS), and total surplus (TS)? (4pts)

**Solution:** Given equilibrium outcome, welfare can be calculated as the area between the curves.

$$
CS = \frac{1}{2}(200 - p^*)q^* = 3200, \\
PS = (p^* - 40)q^* = 6400, \\
TS = CS + PS = 9600.
$$
c. A new competitor, Tyfft, entered this year, with cost per ride of $c_T = 60$. Suber has been in the habit of announcing at exactly at 6am each day the actual number of $q_s$ of rides they will make available that day. What is Tyfft’s best response function, $BR_T(q_S)$? (4pts)

**Solution:** Assume perfect substitutability and the demand curve is $p = 200 - (q_S + q_T)$. Tyfft’s maximizes profit

$$\max_{q_T} (200 - q_S - q_T)q_T - 60q_T,$$

taking $q_S$ as given. The first-order condition is

$$200 - q_S - 2q^*_T(q_S) = 60.$$

Thus, best response function is

$$BR_T(q_S) \equiv q^*_T(q_S) = 70 - \frac{1}{2}(q_S).$$

d. Suber now chooses $q_S$ to maximize its own profit, taking into account Tyfft’s response. Predict the resulting price, number of rides $Q = q_S + q_T$, profits, $CS$, $PS$, and $TS$. (4pts)

**Solution:** Given the best response function of Tyfft, Suber maximizes profit

$$\max_{q_S} (200 - q_S - (70 - \frac{1}{2}q_S))q_S - 40q_S.$$

The first-order condition is

$$130 - q^*_S = 40.$$

Thus, $q^*_S = 90$ and $q^*_T = 70 - \frac{1}{2}(q^*_S) = 25$, $Q^* = q^*_S + q^*_T = 115$, and $p^* = 200 - Q^* = 85$.

Assuming that fixed costs are zero, profits are $(p^* - c_i)q^*(i)$ for $i = \{S, T\}$. So
they are 45(90)=4050 and 25(25)=625 for S and T respectively. Welfare is

\[ CS = \frac{1}{2}(200 - p^*)Q^* = 6612.5, \]
\[ PS_S = (p^* - c_S)q^*_S = 4050, \]
\[ PS_T = (p^* - c_T)q^*_T = 625, \]
\[ TS = CS + PS_S + PS_T = 11287.5. \]

e. Tyfft is considering announcing its own number of rides \( q_T \) simultaneously at 6am each day, and has asked you to predict the impact on its own profit and on customer well-being (CS). What is your prediction? (6pts) [For extra credit, also predict Suber’s profit, PS and TS.]

**Solution:** Both firms take other firm’s strategy as given and maximizes profit. We already solve the same problem for Tyfft in part c). The problem for Suber is different only in marginal cost. Thus, first-order conditions are

\[ 200 - q_S - 2q^*_T(q_S) = 60, \]
\[ 200 - 2q^*_S(q_T) - q_T = 40. \]

The Nash equilibrium \((q^*_S, q^*_T)\) will satisfy both equations above. Thus, Nash equilibrium is \((q^*_S, q^*_T) = (60, 40).\)

CS is \( \frac{1}{2}(200 - p^*)Q^* = \frac{1}{2}(200 - 100)100 = 5,000.\)
Profit of Suber is \((p^* - c_S)q^*_S = (100 - 40)60 = 3,600.\)
Profit of Tyfft is \((100 - 60)40 = 1,600.\)

TS is \( CS + PS_S + PS_T = 5,000 + 3,600 + 1,600 = 10,200.\)

So this policy would increase Tyfft’s profit but reduce CS and TS.

5. Aadvil Adventures (AA) seeks your advice on whether to expand operations into South Stanistan (SS). To do so will involve an immediate sunk cost of 10. If SS’s current regime survives (which you believe will happen with probability 0.4), then the expansion will provide a net revenue stream starting next year of 2 per year for a very long time (approximately forever). However, with probability 0.6, a hostile regime will emerge soon in which cases net revenue will be zero forever.
a. If AA’s opportunity cost of funds is $k = 0.10$, what is the expected present value (EPV) of the expansion? Do you advise expanding or not? Hint: PV of the stream $(0, 1, 1, 1, \cdots)$ is $1/k$. (4pts)

**Solution:** Assume risk-neutral agents. Then optimal decision depends on present discounted value of expected payoffs.

\[
P V = -10 + 0.4 \left\{ \frac{2}{1+k} + \frac{2}{(1+k)^2} + \cdots \right\} + 0.6 \left\{ \frac{0}{1+k} + \frac{0}{(1+k)^2} + \cdots \right\}
\]

\[
= -10 + 0.4 \left( \frac{2}{0.1} \right) = -2 < 0,
\]

so you would advise against expansion.

b. You can hire a political pundit who has 70% accuracy in predicting trends in the Stanistans. (E.g., when a hostile regime actually emerges, then she will have already predicted it with with probability 0.7, etc.) What is your updated belief about a hostile regime if she predicts that current regime will survive? If she predicts that it will not survive? (4pts)

**Solution:** Denote true state as $S_t = \{S, NS\}$ and message as $m = \{s, ns\}$. By Bayes’ theorem, posterior probability $P(NS|s)$ and $P(NS|ns)$ are

\[
P(NS|s) = \frac{P(s|NS)P(NS)}{p(s|S)P(S) + p(s|NS)P(NS)} = \frac{0.3(0.6)}{0.7(0.4) + 0.3(0.6)} \approx 0.39.
\]

\[
P(NS|ns) = \frac{P(ns|NS)P(NS)}{p(ns|S)P(S) + p(ns|NS)P(NS)} = \frac{0.7(0.6)}{0.3(0.4) + 0.7(0.6)} \approx 0.78.
\]

For later use, message probabilities are $p(s) = 0.46$ and $p(ns) = 0.54$, which are given in the denominator of the above expressions.

c. Draw and solve the decision tree when the pundit’s advice is free. (6pts) [For extra credit, compute the economic value of (i.e., the maximum you would recommend that AA pay for) the pundit’s advice?]
6. Developments in Silicon Valley increase productivity worldwide. Other things (i.e., thrift unaffected), what should be the impact on the real interest rate? Nominal interest rates? Current and future real consumption? Wealth? Hint: For a representative agent, draw an old PPF and a new PPF that has higher MROI everywhere. Use geometric and/or calculus arguments to derive your conclusions, mentioning standard assumptions that you use. (10pts)

**Solution:** The increase in productivity shifts PPF upward as in the left panel, where the slope is steeper above each $q_0$. Therefore, at the old interest rate, the
optimal $q_0$ decreases (this is the production effect) and the optimal $c_0$ increases. As shown in the left panel of the figure below, borrowing exceeds lending. In equilibrium, therefore, the real interest rate must rise as in the right panel. The substitution effect decreases $c_0$ and increases $c_1$. Income effect (via the production effect in the left panel) increases both $c_0$ and $c_1$. Overall, $c_1$ increases but the effect on $c_0$ is ambiguous. Although real interest rate increases, $q_1$ also increases and thus effect on real wealth is ambiguous. In the graph, $c_0$ barely increases and real wealth also barely increases. If the inflation rate unaffected, then the nominal interest rate increases by the same amount as the real interest rate.

![Diagram](image)

7. Two companies produce standard tractors, Centipede (C) and Dearjohn (D), at respective marginal cost $c_C = 2$, and $c_D = 3$. When their outputs are $q_C$ and $q_D$, they experience inverse demand $p_C = 14 - 2q_C - q_D$, and $p_D = 15 - 3q_D - q_C$ respectively.

a. what can you infer about the substitutability (or complementability) of the two kinds of standard tractors (e.g., are they perfect substitutes)? (4pts)

**Solution:** They are substitute because when quantity in one product increases, price decreases in the other market through decrease in demand. They are imperfect substitute because increase in quantity in the other market affects less than increase in quantity in own market.
b. Use Nash equilibrium to predict prices and quantities when both companies independently choose output quantities. (8pts)

Solution: C’s problem is to maximize profit

$$\max_{q_C} (14 - 2q_C - q_D)q_C - 2q_C.$$  

The first-order condition is

$$14 - 4q^*_C - q_D = 2.$$  

In a similar way, the first-order condition for D is

$$15 - 6q^*_D - q_C = 3.$$  

This leads to $$(q^*_C, q^*_D) = \left(\frac{60}{23}, \frac{36}{23}\right)$$ and $$(p^*_C, p^*_D) = \left(\frac{166}{23}, \frac{177}{23}\right).$$

c. What is C’s conjectural variation in your Nash equilibrium calculation? What is the actual variation, according to experienced inverse demand? (4pts)

Solution: Assumed conjectural variation is 0 because Nash equilibrium is derived under the assumption that each agent takes rivals’ strategy as given. By rewriting the first-order condition in part b) yields

$$BR_D(q_C) = 2 - \frac{1}{6}q_C.$$  

Thus, actual conjectural variation is $$-\frac{1}{6}.$$