Homework 1

Michał Dereziński *Evolutionary Game Theory*

1. Spreadsheet attached separately.

2. (a) It may only at the beginning. But, since fitnesses are fixed, the best strategy (or strategies) always wins in the end, so the population converges.

(b) The population converges once the dominant strategies take over all the shares.

(c) If there is a unique maximum fitness with non-zero initial share, the steady state is in the corner. If there is two best strategies with equal fitness (and non-zero initial shares), then the steady state is on the face of the simplex corresponding to those strategies. If all three strategies have equal fitness, then the initial state is the steady state, so it may be in the interior.

3. Memes are a concept used to create evolutionary models in situations where there is no physical genes to analyze. This gives us the freedom to come up with all kinds of processes that mimic evolution. The procedure of recombination could certainly be introduced into such a model, as well as the diploid structure, although one may ask whether or not there are real-life environments where such models would be applicable. This seems likely for recombination. The diploid model appears to be more specific to genes themselves, but it is certainly conceivable to use it for memes as well.

4. (a) Suppose that we have $M_{Aa} > 0 > m_{aa}$. Then, we can find the unique interior steady state by solving

$$0 = \frac{\partial \bar{W}}{\partial s_a} = -2m_{Aa} + (4m_{Aa} - 2m_{aa})s_a$$
$$s_a^* = \frac{m_{Aa}}{2m_{Aa} - m_{aa}} \in (0,1)$$

(b) Using the spreadsheet provided by the instructor, manipulating the initial shares and fitness values I verified that:

- if the initial share values are set to the interior steady state, then they stay constant throughout the simulation (for both over- and underdominance).
- For overdominance, setting the initial shares to any interior point guarantees convergence to the interior steady state.
- For underdominance, setting the initial shares even minimally away from the interior causes the shares to converge to a corner steady state.

5.

$$\begin{aligned}
s'_A &= \frac{s_A^2 + \frac{1}{2}(2s_A s_a - m_{Aa}2s_A s_a)}{\bar{W}} \\
\Delta s_A = s'_A - s_A &= \frac{s_A^2 + \frac{1}{2}(2s_A s_a - m_{Aa}2s_A s_a)}{\bar{W}} - \frac{s_A(1 - 2s_A s_a m_{Aa} - s_a^2 m_{aa})}{\bar{W}} \\
&= s_A \frac{(s_A + s_a - m_{Aa}s_a - 1 + 2s_A s_a m_{Aa} + s_a^2 m_{aa})}{\bar{W}} = s_A \frac{(-m_{Aa}s_a + 2s_A s_a m_{Aa} + s_a^2 m_{aa})}{\bar{W}} \\
&= s_A s_a \frac{(-m_{Aa} + 2s_A m_{Aa} + s_a m_{aa})}{\bar{W}} = \frac{s_A s_a}{2\bar{W}} \frac{\partial \bar{W}}{\partial s_A}.
\end{aligned}$$

6.

$$\frac{\dot{s}_i}{s_i} = \frac{1}{s_i} \frac{\dot{N}_i}{N_T} = \frac{\dot{N}_i}{s_i N_T} = \frac{\dot{N}_i}{N_i}.$$