Problem Set 4

Econ 200

Q1. As a financial analyst for a biotech startup company, you are to advise on whether to pursue a line of research that will cost \$10 (R&D) now and with probability 0.3 will lead to a potential product. It costs \$50 to test such a potential product, and the test leads to FDA approval (validation) with probability 0.4. At that point the product is marketable, and at a further cost of \$50 (production) it can be sold, bringing on average \$200 in revenue (net of other costs not mentioned). Assume that all dollar figures are present values in \$millions. Revenues come only from marketable products.

a. Draw and solve the decision tree, assuming that you must sink all mentioned costs (R&D, validation and production) immediately.

b. Redraw and solve the decision tree assuming that you can wait to see the R&D outcome before sinking the validation cost, and can wait for FDA approval before sinking the production cost.

c. Now assume that the situation is as in b. above except that with probability 0.2 a marketable product is a blockbuster, and the realization (blockbuster vs ordinary) will be known right after FDA approval. At an additional cost of \$100 (expansion) a blockbuster generates net revenue of 1000 instead of the ordinary 200. Once more, draw and solve the decision tree. Also note the probability that the line of research will ultimately produce a blockbuster.

Solution: This is a standard question on finite-period decision-making. We draw decision trees and solve them by backward induction. Once you draw a tree correctly, you simply have to calculate expected value of the lottery when you see a nature node and you simply have to choose the decision with highest payoff when you see a decision node.

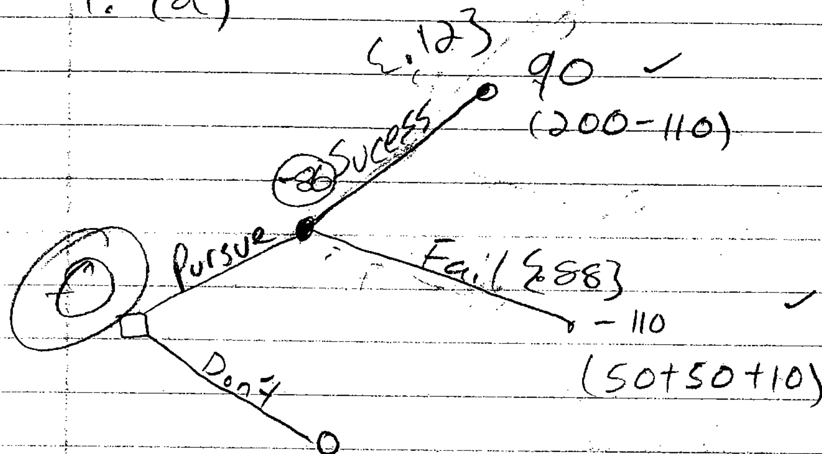
How to draw a decision tree correctly requires a bit of practice, but what is important is that once you draw a tree, you have to be able to check whether the situation expressed in the question is properly represented in the tree or not. If the tree does not represent the situation correctly, think about which part of the tree you have to modify. Repeat this process until the tree represents the situation properly.

F 2005

Answer Key - Final Exam

I)

1. (a)

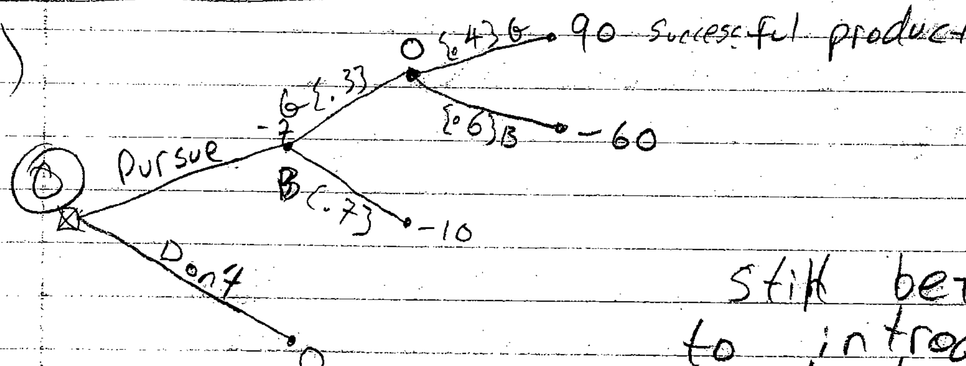


3/3

Don't Pursue -.86 < 0

(b)

G = Good
B = Bad

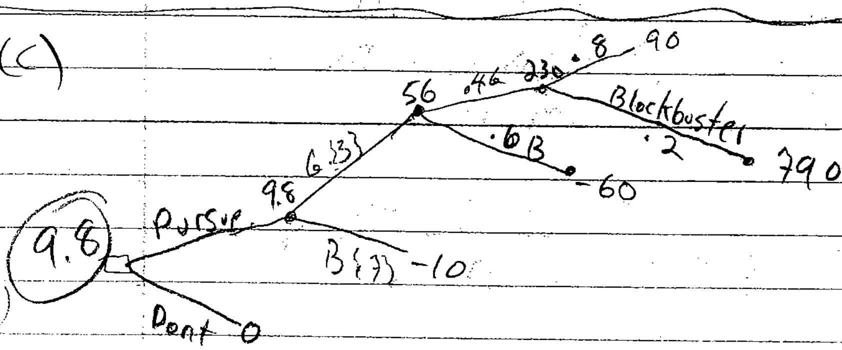


4/4

still better not to introduce the product

-.7 < 0

(c)



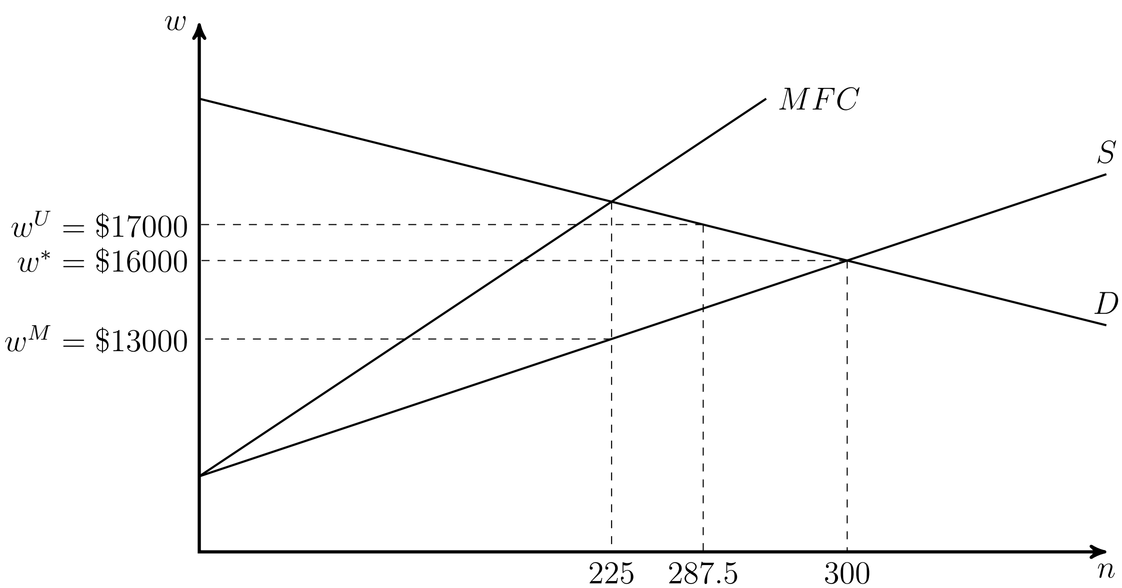
you should pursue the product since expected payoff is now 9.8

chance of blockbuster = .3 · .4 · .2 = .024 = 2.4%

Q2. The demand for TAs at the University of Chico is approximated by the inverse demand function $w = 40000 - 80n$, where w is the annualized wage and n is the number of TAs hired. The supply is approximated by $w = 4000 + 40n$. If U Chico uses its market power fully, what wage will it pay and how many TAs will it hire? If TAs unionize and enforce $w = \$17,000$, how many TAs will be hired? Comment briefly on the efficiency (TS=CS+PS) implications of the union wage.

Solution: The university chooses the number of TAs to maximizes its objective function, which is the benefit from TAs minus cost to hire. First-order condition is marginal benefit equals marginal factor cost. Total cost is given by $(4000 + 40n)n$, so marginal factor cost is $w = 4000 + 80n$. Setting marginal factor cost equal to inverse demand function, the university hires $n^M = 225$ TAs and sets wages equal to $w^M = \$13,000$ when it acts as a monopsonist. When students unionize and demand wages of $w^U = \$17,000$, the university will hire $n^U = 287.5$ TAs.

When the TAs unionize, the effect is that there is a price floor on wages. When we looked at price floors earlier in the quarter, we saw that they resulted in deadweight loss. But in this problem, the price floor actually *reduces* deadweight loss since it brings us closer to competitive equilibrium. This is because the monopsonist is already inefficient, so introducing a second distortion such as a price floor can actually reduce inefficiency. If the union compromises and sets the wage $16,000, this achieves the most efficient allocation.



Q3. Baytech sells gizmos in the home market where it faces the demand function $q_H = 100 - 3p_H$. It also sells the same product in a foreign market where it faces the demand function $q_F = 200 - 7p_F$. Its cost function is $c(q_T) = 30 + 12q_T + q_T^2$, where $q_T = q_H + q_F$.
(a) What output and price choices maximize Baytech's profit?

Solution: Write profit as

$$\text{TR}_H(q_H) + \text{TR}_F(q_F) - c(q_H + q_F)$$

where TR_H and TR_F are total revenues in the home and foreign markets, respectively. By taking derivatives, we see that Baytech's profit-maximizing conditions are

$$\text{MR}_H(q_H) = c'(q_H + q_F) = \text{MR}_F(q_F)$$

This says that marginal revenue at home equals marginal revenue abroad equals the marginal cost of producing for the combined market. (By the way, $\frac{dc(q_T)}{dq_H} = \frac{dc(q_T)}{dq_T} \cdot \frac{dq_T}{dq_H} = \frac{dc(q_T)}{dq_T} = c'(q_H + q_F)$ since $\frac{dq_T}{dq_H} = 1$. Repeat this to show that $\frac{dc(q_T)}{dq_F} = c'(q_H + q_F)$.)

Substituting in the marginal revenues (obtained by taking the derivative of $\text{TR}_H(q_H) = p_H(q_H) \cdot q_H$ and $\text{TR}_F(q_F) = p_F(q_F) \cdot q_F$) and marginal cost,

$$\frac{100}{3} - \frac{2}{3}q_H = 12 + 2(q_H + q_F)$$
$$\frac{200}{7} - \frac{2}{7}q_F = 12 + 2(q_H + q_F)$$

This system of 2 equations in 2 unknowns can be solved to obtain $q_H = 7.45$ and $q_F = 0.73$ which gives the prices $p_H = 30.85$ and $p_F = 28.47$.

(b) Antidumping regulations force Baytech to unify its prices. What choice of $p = p_H = p_F$ now maximizes profit? What is the maximum Baytech would rationally spend to eliminate the antidumping regulation?

Solution: With $p = p_H = p_F$, we now have $q_T(p) = q_H(p) + q_F(p) = 300 - 10p$ which gives inverse demand $p(q_T) = 30 - \frac{1}{10}q_T$. Baytech's optimization problem is now

$$\max_{q_T \geq 0} \quad \left(30 - \frac{1}{10}q_T\right) q_T - (30 + 12q_T + q_T^2) \quad \text{such that } q_H \geq 0, q_F \geq 0, q_H + q_F = q_T$$

The first-order condition gives $q_T = 8.18$ and $p = 29.18$. Remember, though, that an interior solution must also satisfy $q_H \geq 0$ and $q_F \geq 0$. It turns out that $q_F(29.18)$ is negative. Therefore, the first-order condition does not gives us an interior solution so we have to check corner solutions.

First, find the solution when $q_F = 0$. The problem is

$$\max_{q_H \geq 0} \quad (\frac{100}{3} - \frac{1}{3}q_H)q_H - (30 + 12q_H + q_H^2)$$

which gives $q_H = 8$, $p = 30.67$, and profits of 55.33. The other possible corner solution is when $q_H = 0$. In this case, the problem is

$$\max_{q_F \geq 0} \quad (\frac{200}{7} - \frac{1}{7}q_F)q_F - (30 + 12q_F + q_F^2)$$

which gives $q_F = 7.25$, $p = 27.535$, and profits of \$30.07.

Therefore, Baytech will choose to only its gizmos in the home market and make a profit of \$55.33. Baytech's unregulated profit in part (a) is \$251.88, implying that Baytech should be willing to spend up to $55.54 - 55.33 = 0.21$, \$0.21 per period to overturn the regulation.

Q4. Econometric analysis of air travel demand yields the following elasticity estimates for (first class, unrestricted coach, discount) demand: income = (1.8, 1.2, 1.1), and own-price = (-0.9, -1.2, -3.7). Predict the fares that a profit-oriented airline would pick for a route with no serious competition and marginal cost \$150 per passenger.

Solution: This is an example of third-degree price discrimination. Applying the mark-up formula $p_i = \frac{|\epsilon_i|}{|\epsilon_i|-1}c$ from the notes, we get

$$\begin{aligned} p_F &= \frac{0.9}{0.9-1}(150) = -\$1350 \\ p_U &= \frac{1.2}{1.2-1}(150) = \$900 \\ p_D &= \frac{3.7}{3.7-1}(150) = \$205.6 \end{aligned}$$

Plainly, the formula does not give a valid price for first-class tickets, p_F. This is because demand for these tickets is inelastic ($|-0.9| < 1$). Faced with inelastic demand for first-class tickets, the profit-maximizing firm will raise p_F to an arbitrarily high level until demand is no longer inelastic. (As long as demand is inelastic, when a firm raises its price total revenue will increase and total costs will fall.)

Q5. Direct demand is $Y = 86 - p$, where Y is the sum of output across all firms, and each firm has cost function $c(y) = 14y$.

(a) Firms operate in the market by setting quantities. What are outputs, prices, profits and deadweight losses for monopoly, duopoly, triopoly and perfect competition markets? Show all work, but then collect your answers into a table, with columns for market structure and rows for performance measures. Which market is most efficient and WHY?

Solution: With perfect competition, the profit maximization condition $p = \text{MC}$ implies that $86 - Y = 14$, or $Y^* = 72$, $p^* = 14$, and $\pi^* = 0$. Of course, with perfect competition there is no deadweight loss.

Rather than solving the cases with 1 firm, 2 firms, and 3 firms separately, it's easier to just solve for N firms. Then, firm i maximizes

$$\max_{y_i} \quad [86 - (y_1 + \ldots + y_N)]y_i - 14y_i$$

which has the first-order condition

$$72 - (y_1 + \ldots + y_{i-1} + 2y_i + y_{i+1} + \ldots + y_N) = 0$$

which can be re-arranged as

$$y_i = 72 - Y$$

Note that this condition holds for each of the N identical firms. If we add all these conditions together, we get

$$\sum_{i=1}^{N} y_i = N(72 - Y)$$

Of course, the sum on the left-hand side is just equal to Y, so we can solve the above for Y to get $Y = 72\frac{N}{N+1}$. Note that when we let N go to infinity, we get $\lim_{N\to\infty} Y = Y^*$.

For the monopolist case where $N = 1$, we get $Y^M = 36$, $p^M = 50$, $\pi^M = 1296$, and deadweight loss of $\frac{1}{2}(72-36)(50-14) = 648$. Note that to find the deadweight loss, we first had to find industry supply. Since each firm has inelastic supply at $p = 14$, inverse industry supply is also the horizontal line $p = 14$.

With a duopoly ($N = 2$), we get $Y^D = 48$, $p^D = 38$, $\sum_{i=1}^{N} \pi_i^D = 1152$, and deadweight loss of $\frac{1}{2}(72-48)(38-14) = 288$. With a triopoly ($N = 3$), we get $Y^T = 54$, $p^T = 32$, $\sum_{i=1}^{N} \pi_i^T = 972$, and deadweight loss of $\frac{1}{2}(72 - 48)(38 - 14) = 162$.

To summarize the results:

	Y	p	$\sum \pi_i$	DWL
Monopoly	36	50	1296	648
Duopoly	48	38	1152	288
Triopoly	54	32	972	162
Perf. Comp.	72	14	0	0

Not surprisingly, perfect competition comes out as the most efficient market structure. With the other market structures, the firm(s) restrict output in order to raise prices and earn profits. This behavior, however, generates deadweight loss.

II. Short essays. Aim for 150-250 words for each essay. Please print on separate pages. 1. Which of these firms are natural or unnatural monopolists? What are reasonable public policies towards them? Google, Facebook, AirBnB, Amazon, Uber, Golden State Warriors.

Solution: Natural monopolies are ones which arise from insufficient demand or high (natural) barriers to entry, among other things. Facebook might be considered a natural monopoly, because it would be extremely difficult to lure new customers away from Facebook to competitors until you have a large group using the competitor (network effects). A reasonable public policy toward a natural monopoly is to enforce anti-trust law to ensure that these companies do not unfairly overcharge customers, undersupply services, or otherwise take advantage of their market power.
Unnatural monopolies arise from policies or interventions that restrict market entry. An example might be the NBA allowing only one team to serve the San Francisco Bay Area, though it could be natural if there is only enough demand for one team. The best policies here are those which remove barriers to entry or otherwise encourage competition.
Suggestion: Google, Facebook, AirBnB, and Uber are natural monopolists due to network externalities. Amazon is a conglomerate, trying to use monopoly power in one area to undercut incumbents in related services. The Warriors are an unnatural monopoly generated by an anti-trust exemption for the NBA. The NBA regulates entry, hiring, draft picks, etc.

2. Pick an industry you know something about; if none, pick bookstores. Use one or two models of imperfect competition to organize a discussion of output and pricing decisions and profitability in the chosen industry. For example, for bookstores, you might consider Bertrand oligopoly (covered in class), or monopolistic competition, or dominant firm/competitive fringe (look them up if you are interested).

Solution: Any relevant, well-argued response is appropriate. Friedman's postion: retail bookstores, before Borders and Amazon, were monopolistic competitors, or faced a kinked demand curve. Now Amazon is a monopolist in most local markets (not Santa Cruz), but is keeping prices low to prevent entry.