Midterm

Econ 200

1. Zabella Inc. creates game apps for smartphones. Their VP hires you to estimate the demand funciton fro their best-selling product, and provides monthly data on its price p_z, the main rival's price p_R, and the mean income M of customer segments. For monthly unit sales volume Z, you obtain the fitted equation

$$\ln Z = 3.1 - 1.2 \ln p_Z + 0.8 \ln p_R + 0.6 \ln M. \tag{1}$$

a) What effect would a 10% increase in customer's mean income have on sales, according to equation (1)? (2pts)

> **Solution:** 1% increase in income increases sales by 0.6%, so 10% increases leads to 6% increase in sales.

b) If Zabella and rivals both raised prices by 10%, what would be the impact on number of units sold, according to equation (1)? (2pts)

> **Solution:** 10% increase in own price decreases sales by 12% and 10% increases in rival price increases sales by 8%, so to total effect will be 4% decrease in sales.

c) In what sense, if any, should Zabella's demand be homogeneous? What are the practical implications, if any, for estimating the demand function? (3pts)

> **Solution:** Demand function must be homogeneous of degree 0 in $(\mathbf{p}, m)$ (doubling all the prices and income does not change the feasible set and thus the solution to optimization problems). In practice, when you estimate equation (1), you assign a condition that the sum of coefficients except constant equals to 0.

2. You also estimate Zabella's cost function, holding constant the salaries of software engineers and the price of cloud storage and other inputs, and obtain

$$C(Z, Y) = 10.2 + 3.1Y + 2.2Z - 0.9YZ \tag{2}$$

as the cost of producing Y units per month of their new game as well as Z units of their best-selling game.

a) Does eq. (2) imply increasing, decreasing or constant returns to scale in production? (2pts)

Solution: The cost when doubling Y, Z is less than the cost of producing (Y, Z) doubled, so it is IRS.

b) Does the equation imply economies of scope? Cost complementarities? Justify your answers briefly. (3pts)

Solution: Yes for both. Economies of scope exists because $C(Z,0) + C(0,Y) - C(Z,Y) = 10.2 + 0.9YZ > 0$. Cost complementarities exists because $\frac{\partial^2 C}{\partial Z \partial Y} = -0.9 < 0$.

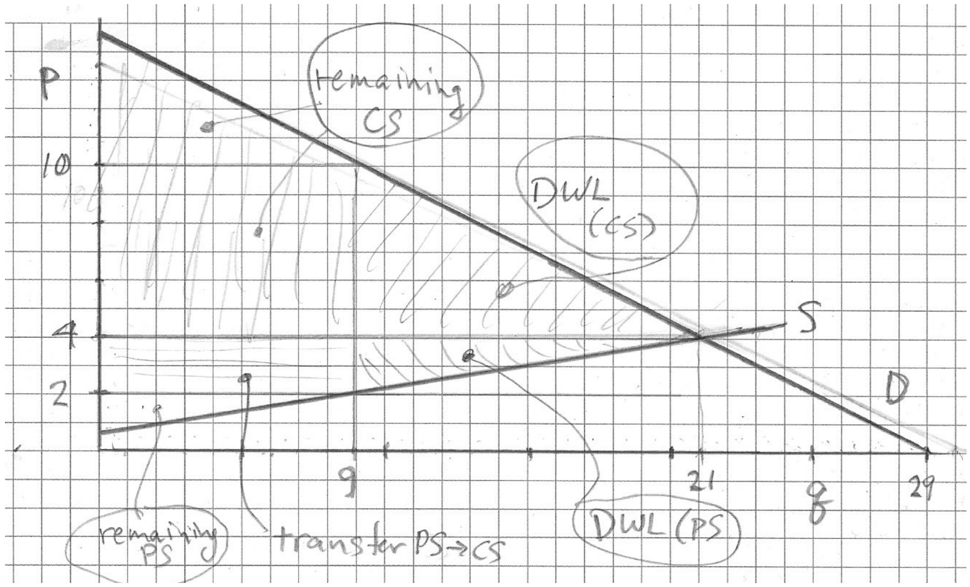
c) If Zabella abandons their new game, what would be the efficient scale for their best-selling game? What dose that imply about long-run competitive equilibrium for that game?

Solution: Assuming the cost structure does not change when the new game is abandoned and also in the long run, average cost is $\frac{C(Z,0)}{Z} = \frac{10.2}{Z} + 2.2$. This is decreasing in Z everywhere if we assume the linear cost function holds everywhere. Thus, efficient scale is infinity and the market won't be competitive equilibrium.

3. Supply of five-gallon plastic water containers in Napa county is approximated by $S(p) = 6p - 3$ over the relevant price range, and demand is usually approximately $D(p) = 9 - 2p$. After the recent fire, however, the quantity demanded jumped by 20 units at each price. (A unit is a thousand containers, and price is quoted per container.)

a) What is the competitive equilibrium price usually? After the recent fire? (4pts)

Solution: Competitive equilibrium price p^* makes $D(p^*) = S(p^*)$. i) Before the fire, $p^* = 1.5$ and ii) after the fire, the demand curve becomes $D(p) = 29 - 2p$ and $p^* = 4$.



b) Public outrage at the price jump convinced stores to impose a price ceiling at $p = 2.00$. Relative to the post-fire competitive equilibrium, what impact do you predict that such a price ceiling will have on consumer surplus? Producer surplus? Deadweight loss? (4pts)

Solution: Competitive equilibrium quantity before the price control was $q^* = 21$ and after the control is limited by supply, to $q^* = S(2) = 9$. When quantity is $D(p) = 9$, the inverse demand function yields demand price $p = 10$. $\Delta CS = 2(9) - \frac{1}{2}(10-4)(21-9) = -18$
$\Delta PS = -\frac{1}{2}(9+21)2 = -30$.
$DWL = \frac{1}{2}(10-2)(21-9) = 48$.

c) How could the price ceiling be enforced? What impact do you expect in the long run?

Solution: There probably will be a black market in water containers, but enforcement and general chaos will probably keep it from clearing at $p = 10$. In longer run, aid agencies will probably increase the supply of containers sufficiently to drive equilibrium price to 2 or lower.

4. Yabin's utility function for x ounces of moisturing lotion and y dollars available to spend on other products is $u(x, y) = y + 4\sqrt{x}$.

a) Compute her Marshallian demand function for x. (5pts)

Solution: Normalize the price of "good" y to 1. Lagrangian of the utility maximization problem is

$$L = y + 4\sqrt{x} + \lambda(m - px - y). \tag{3}$$

The FOCs (assuming $y > 0$) are

$$1 = \lambda, \tag{4}$$

$$4\frac{1}{2}x^{-\frac{1}{2}} = \lambda p. \tag{5}$$

This leads to the standard tangency condition. Marshallian demand is

$$x^* = 4p^{-2}. \tag{6}$$

One special feature about quasi-linear utility function is that demand for x depends only on own price.

b) What is her income elasticity for x? Own-price elasticity? Cross-price elasticity? (5pts)

Solution: Taking log of the Marshallian demand, Own-price elasticity is -2, income elasticity is 0, and cross-price elasticity is undefined since the price of y is normalized to 1.0.

Solution: In the solution above (and in standard use of the quasi-linear demand function), it is assumed that m is large enough that $y > 0$. When $m < \frac{4}{p}$ and borrowing ($y < 0$) is not allowed, then Marshallian demand is $x^* = \frac{m}{p}$. In this case, own-price elasticity is -1 and income elasticity is 1.

5. Suppose that you estimate the following cost function:

$$c(y, w_1, w_2) = y^{1.1}[2w_1^{0.25} + w_2^{0.25}]^{4.0} \tag{7}$$

where c=total cost, y=output quantity, and the w_i's are prices of key inputs.

a) What is the marginal cost function implied by the equation when input prices are $w_1 = 16$ and $w_2 = 81$.

Solution: Marginal cost is

$$\frac{\partial c}{\partial y} = 1.1y^{0.1}[2w_1^{0.25} + w_2^{0.25}]^{4.0}. \tag{8}$$

When $w_1 = 16$ and $w_2 = 81$,

$$MC = 1.1y^{0.1}[2(2) + 3]^4 = 1.1y^{0.1}7^4. \tag{9}$$

b) What can you say about returns to scale of the underlying production function? (2pts)

Solution: Since the marginal cost is increasing in y, the production function is DRS.

c) What can you say about how the output depends on the inputs (e.g., that is Cobb-Douglas with particular exponents)? (2pts)

Solution: Since the cost function is CES, the production function is also CES by duality. The parameter ρ in the production function

$$f(x_1, x_2) = [(a_1x_1)^\rho + (a_2x_2^\rho)]^{\frac{\alpha}{\rho}} \tag{10}$$

satisfies $\frac{1}{\rho} + \frac{1}{0.25} = 1$. That is, $\rho = -\frac{1}{3}$.

d) What main properties should cost functions have? Check whether or not the cost function above has each of these properties (3pts)

Solution: i) Homogeneous of degree 1 in $\mathbf{w}$. Yes.

$$\begin{aligned} c(\lambda\mathbf{w}, y) &= y^{1.1}[2(\lambda w_1)^{0.25} + (\lambda w_2)^{0.25}]^4 \\ &= y^{1.1}[\lambda^{0.25}(2w_1^{0.25} + w_2^{0.25})]^4 \\ &= \lambda c(\mathbf{w}, y). \end{aligned} \tag{11}$$

ii) Positive for all positive input prices and output. Yes, because addition, multiplication and taking positive powers preserve positivity.
iii) Continuous in $\mathbf{w}$. Yes, because it is differentiable.
iii) Nondecreasing in ($\mathbf{w}$,y). Yes, by looking at derivative.
iv) Concave in $\mathbf{w}$. Yes, if you try but we do not expect to do this in exams

e) Write down conditional input demand functions implied by the above cost function. (4pts)

Solution: By Shepard's lemma,

$$x_1^*(y, \mathbf{w}) = \frac{\partial c}{\partial w_1} = 2y^{1.1}[2w_1^{0.25} + w_2^{0.25}]^3 w_1^{-\frac{3}{4}} \tag{12}$$

$$x_2^*(y, \mathbf{w}) = \frac{\partial c}{\partial w_1} = y^{1.1}[2w_1^{0.25} + w_2^{0.25}]^3 w_2^{-\frac{3}{4}}. \tag{13}$$